Building a Credit Risk Valuation Framework for Loan Instruments

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We present a general option-valuation framework for loans that provides valuation information at loan origination and supports mark-to-market analysis, portfolio credit risk and asset and liability management for the entire portfolio. We describe, in detail, the main structures found in commercial loans and the practical assumptions required to model the state-contingent cash flows resulting from these structures. The characteristics of the credit risk model necessary to capture the main features of the problem are described. Finally, we discuss the families of credit models appropriate for pricing, the data required for their calibration and reasonable criteria for choosing the sophistication of the model. We propose a multi-state, ratings-based credit model with three credit drivers: the credit state of the obligor, the level of risk-free rates and the spreads. Though we focus primarily on large corporate and middle-market loans, the approach is applicable more generally to bonds and credit derivatives.

Since its application to derivatives valuation in the early 1970s, no-arbitrage pricing has become the basis for managing the risk of the trading and investment books of financial institutions. No-arbitrage techniques are used to price and hedge securities such as bonds and derivatives, to mark-to-market (MtM) portfolios and to measure risk.

The application of option valuation techniques to bank loans has been much slower in developing. Most banks today manage the credit risk of their loan books in fairly simple and basically static ways. Perhaps the most prevalent method for pricing and managing loans applies the concept of RAROC (risk-adjusted return on capital). The RAROC approach attempts to distribute aggregate risk costs down to businesses, products customers and, ultimately, individual transactions. Measures of static, marginal risk contributions are used in the RAROC approach to allocate capital costs directly to individual loans in relation to the firm's aggregate debt and equity costs.

Since RAROC is not a "no-arbitrage" technique, it does not reconcile the prices of loans with those of similar securities available in the market (such as bonds, other loans and credit derivatives). Hence, it cannot assess comparative business opportunities and arbitrage-like situations arising from relative price mismatches. In addition, it is unable to capture the natural hedges that often motivate the creation of new credit securities. Finally, while several of the financial principles behind RAROC seem generally sound, there are many limitations in its implementation, as has been pointed out in the literature (Shearer and Forest 1998). For example, the approach neglects the state contingency of many loan cash flows, takes a static view of credit risk, generally considers an arbitrary fixed horizon in pricing credit risk and uses highly subjective parameters in practice.

Many financial institutions today are considering a move towards mark-to-market approaches for managing their traditional lending business. An MtM approach for loans can facilitate better pricing and structuring of credit risky instruments, more flexible and dynamic management of credit portfolios and greater exploitation of arbitrage opportunities. With wholesale bank loans, corporate bonds and credit derivatives together accounting for more than $30 trillion (all amounts is USD) in exposures worldwide, better valuation and risk-management techniques hold the potential for enormous business benefits. Those who stand to benefit the most are the institutions that take advantage of an MtM approach to understand the effects of structure and embedded optionality on the value of credit instruments.

We present a general option-valuation framework for loans. While we focus primarily on large corporate and middle-market loans, the approach is applicable more generally to bonds and credit derivatives. This framework provides key valuation information during loan origination, and it supports MtM analysis, as well as the portfolio credit risk and asset and liability management functions.

We emphasize the modelling of key product-specific features of loans and not the simple application of a specific type of pricing model to the problem. To make an effective choice of underlying credit risk model with broad applicability, one must understand these features of loans and have an informed practical view of the market and the data available. While this may seem obvious from a practitioner perspective, most of the academic literature has steered clear of many of the complications of loan structures. Instead, many papers focus on building new and improved credit risk pricing models, and illustrate their applications with simple instruments such as straight bonds and simple credit derivatives, thereby avoiding many of the details needed in practice, (e.g., Jarrow and Turnbull 1995; Jarrow et al. 1997; Madan and Unal 1998; Jarrow and Turnbull 2000). These papers offer no solutions to practitioners choosing and adapting these models to price their generally complex credit instruments, and calibrating them to available data.

One can readily find articles and books describing the features of credit derivatives and their application (e.g., Das 1998; Tavakoli 1998). However, it is more difficult to find work that describes the structures of loans, their embedded optionality, the data available for pricing these assets and the choice of appropriate pricing models. Early research in this type of application was performed at Citibank (Asarnow 1994; Ginzburg et al. 1994) and was continued by Aguais et al. (1998) and Aguais and Santomero (1998). While our discussion falls short of a comprehensive survey of loan instruments, we present a framework that incorporates several main structures encountered in practice and describes a consistent approach to modelling the underlying risk factor processes.

We lay a tripartite foundation to motivate the general valuation framework:

- **The main structures** found in commercial loans, such as utilization of credit lines and options to prepay. We describe these structures and outline the practical assumptions required to model the resulting state-contingent cash flows.
- **The credit model characteristics** that are necessary to capture the main features of the problem. Three factors are generally required to model the state contingency of cash flows in a reasonable way. These three factors explain the creditworthiness of the borrower, the level of risk-free interest rates and the level of credit spreads. All three factors can, in principle, be stochastic.
- **The families of pricing models and the data required for their estimation.** We discuss existing credit models that are appropriate for these problems as well as reasonable criteria for choosing the model based on a trade-off between speed, complexity, data availability and accuracy.

The rest of the paper is organized as follows. The next section serves as background, describing briefly why option-valuation techniques are gaining practical status and acceptance for loan portfolios. Thereafter, we present examples of loan structures and describe the optionality embedded in these structures. The following section describes various models that capture these structures and the rationale for several economic and behavioral assumptions. Following a brief discussion of the characteristics of appropriate credit risk pricing models, we motivate the use of various families of models, outline the data required and discuss practical limitations. As a result, we describe a general framework for implementing these models.

Applying option valuation techniques to credit risk

While the application of option valuation to securities with underlying credit risk was originally envisioned by Merton over 25 years ago (Merton 1974), it was only in the late 1980s that credit risk option-valuation models began to appear in applications. Three main factors contributed to this long delay. First, credit risk modelling is complex and, hence, has trailed behind that of market risk (including equities, foreign exchange and risk-free interest rates). Second, many have accepted the pessimistic view that the standard assumptions made for tractability in no-arbitrage models (such as continuous trading, complete markets, no-frictions and the like) generally do not apply when valuing credit risky instruments. Finally, consistent with this view, financial institutions have, by and large, opted for static management of their (illiquid) credit risks.

Financial institutions, however, are being forced to reconsider these practices and move towards MtM approaches for managing their bank loans for several reasons:

- **Evolution of credit risk markets.** The 1990s saw the development of stronger bond markets, secondary loan markets and a tendency for these two markets to converge. Furthermore, the credit derivatives industry has burgeoned, resulting in enhanced liquidity to support the needs of market participants to transfer credit risk.
- **Advances in credit risk models.** Several decades of research have resulted in a better understanding of the nature of credit risk and in various practical pricing and risk-management models that can be calibrated to observable prices and historical data.
- **Integration of market and credit risk**. The advent of credit derivatives to support the transfer of credit risk and the convergence of credit markets are compelling financial institutions to manage the risk in the banking and trading books in a more unified manner. The assumption that credit risk can not be traded actively is being reconsidered, and the application of no-arbitrage models seems more realistic. This trend has led also to the development of pricing and portfolio models that integrate market and credit risk (e.g., Das and Tufano 1996; Jarrow and Turnbull 2000; Iscoe et al. 1999).
- **Trends in regulation and best practices.** Although a market-based valuation and assessment of credit risk is not yet required, both regulatory trends and best practices point in that direction in the long term. This is evident from the proposal of the Bank for International Settlements (BIS 1999) to amend the regulatory regime and the various discussion papers that have appeared in response to it, as well as the disclosure of loan MtM practices by several institutions.
- **Improvements in technology.** The advent of computational technology provides ready access to non-traditional institutions and investors in the credit markets, and allows the application and delivery of more sophisticated computational tools to price and manage credit risk. Furthermore, the availability of internet tools provides an effective means to distribute on-line credit information and valuation tools to a large number of users.

Common types of credit instruments

The vast majority of credit instruments involve a mixture of standard types that lend themselves to a rather straightforward specification. These types include

- bond
- term loan
- revolver
- financial letter of credit
- banker's acceptance
- default swap
- total return swap
- multi-option facility.

General descriptions of these standard credit instruments appear in Appendix 1.

To illustrate the state contingency embedded in credit instruments, we provide four examples.

Large corporate term loan

Consider a recent syndicated deal of $115 million to help fund the acquisition of PlayCore Holdings Inc., an unrated holding company with interests in the sporting and games industries. The agreement closed on April 14, 2000.

The deal includes a $30 million revolver, a $25 million term loan A and a $60 million term loan B. Credit is secured by a borrowing base composed of 85% of eligible accounts receivable, 60% of eligible inventories, plus $3,000 monthly from November through March. Covenants require, among other things, hedging of some interest rate risk, maintenance of minimum fixed-charge coverage ratios, limitations on dividends, and use of excess cash flow, debt or equity issuance, or insurance proceeds to retire outstanding credit under this agreement. Pricing is tied to the ratio of funded debt to EBITDA (earnings before interest, taxes, depreciation and amortization). In default, pricing increases by 200 basis points (bps). The contract allows prepayment without penalty at any repricing date.

We describe the term-loan B component, which is marketed to loan funds. The final maturity of the loan is July 1, 2006—87 months after the April 14, 2000 closing. The 20 quarterly payments of $150,000, starting on October 1, 2000, are followed by eight quarterly payments of $7,125. The loan amortizes over several quarters. Initially, at contract closing, this facility is priced at PRIME + 225bps or LIBOR + 400bps. Thereafter, the pricing grid, summarized in Table 1, determines pricing on the basis of the company's ratio of indebtedness to cash flow as shown in the most recent financial statement. The current pricing corresponds to a ratio between 4.25 and 4.75, as shown in the second row of Table 1.

Level	Debt to cash flow ratio	Prime + (bps)	LIBOR + (bps)
1	4.75 or greater	250	425
2	[4.25, 4.75)	225	400
3	less than 4.25	200	375

Table 1: Pricing grid of PlayCore term loan B (LPC Gold Sheets 2000a)

Large corporate revolving line

Consider a piece of another recent syndication. The entire $150 million package closed on March 29, 2000; final maturity is March 29, 2003. It provides working capital for Rollins Truck Leasing, which is a BBB+ rated company in the truck rental and leasing business.

The deal includes two $75 million revolvers, one with a 364-day term and the other with a three-year term. Credit is secured by 90% of the net equipment value of all motor vehicle equipment. Covenants include a maximum ratio of funded debt to adjusted tangible net worth and material restrictions on dividends. Pricing is tied to the company's senior debt rating. In default, pricing steps up by 200bps or to PRIME + 200bps, whichever is greater. The agreement includes a letter of credit (LC) option. The contract allows prepayment without penalty at any repricing date.

We describe the three-year facility. The commitment is a bullet bond that expires in its entirety at term. Through September 30, 2000, this loan has a price of Libor + 75bps, a commitment fee (CF_{CF}) of 17.5bps annually, and a letter of credit fee (CF_{LC})of 12.5bps at issuance plus 75bps annually. Thereafter, beginning on the date set by the contract, the grid, summarized in Table 2, sets pricing on the basis of the company's most recent senior unsecured debt rating established by Standard and Poor's (S&P). Thus, for example, if the company is downgraded to BBB, the loan moves to Libor + 95bps, a commitment fee of 20bps annually, and a letter of credit fee of 12.5bps annually, as given in the third row of Table 2.

Level	Senior rating	Prime + (bps)	LIBOR + (bps)	Commitment fee, CF (bps)	Letter of credit, LC (bps)
1	A– or better	0	60	15.0	12.5 + 60
2	[BBB+, A–)	0	75	17.5	12.5 + 75
3	[BBB, BBB+)	0	95	20.0	12.5 + 95
4	worse than BBB	0	115	25.0	12.5 + 115

Table 2: Pricing grid of Rollins' 36-month revolver (LPC Gold Sheets 2000b)

Middle-market revolving line

Relatively few of the larger middle-market loans involve syndicates. As is typical of middle-market loans today, the term is shorter and the structure simpler than most large corporate instruments. However, middle-market loans are becoming more complex, and some of the larger ones now include three- to seven-year terms, commitment fees and pricing grids.

To illustrate a typical middle-market loan, we consider a bilateral deal involving one bank but, for confidentiality, change the borrower's name and some of the less important details of the agreement. This one-year $8.5 million revolving line supports the working capital needs of NE Timber, which is in the logging business. According to the bank's internal credit rating system, NE has an S&P-equivalent rating of B.

A borrowing base composed of cash plus 80% of zero- to 90-day receivables, plus 60% of inventory, plus 40% of raw timber secures credit under the agreement. The contract includes the standard covenant package, which limits dividends, requires maintenance of a minimum ratio of operating cash flow to debt, and prescribes that any new debt be used first to retire credit under this agreement. Pricing is at PRIME + 250, with no fee on unused amounts, as summarized in Table 3.

Price level	Prime + (bps)
1	250

Table 3: Pricing grid of NE's line of credit

Prepayment involves no penalty at repricing dates, and a 2% hedge-breakage fee at other times. The loan matures 12 months after closing. The commitment amortization is a bullet bond.

Credit-default swap

Consider an agreement providing protection against default by a Latin American country. Under the terms of the five-year contract, the protection buyer owes a fee of 250bps per annum payable quarterly, in advance, on a notional principal of $25 million. The protection seller owes nothing unless the country defaults, with default defined by standard documentation. Broadly, default occurs if the Latin American country misses a senior debt payment or offers a distressed exchange of assets, or if the market value of the underlying asset identified in the contract falls by more than a specified amount.

In case of default, the protection seller pays par for $25 million at face value of the underlying USD denominated asset, if available, to the protection buyer. Alternatively, if the underlying asset is unavailable, a fair net cash settlement is paid as determined by the calculation agent identified in the contract. The contract terminates within a specified short period following default, and the protection buyer has the right to cancel the agreement at any time.

Cash-flow timing and components

As illustrated above, the embedded options and other features characteristic of most credit agreements cause the associated cash flows to vary over time and with changes in the state of the world necessitating a cash-flow modelling approach that accounts for state dependency. We start by describing the calculation of cash flows at a given state and time, then discuss the modelling of credit-line usage and prepayment behavior. These features substantially affect the cash flows and the value of credit contracts at each state and time.

We assume a series of discrete time steps though, for ease of exposition, we focus on a single time step. All the contingent cash flows whose contractual values depend on the state at the beginning of the time step are modelled. However, payments may occur either at the beginning or the end of the time step.

The cash flows realized depend on certain contingencies:

- The payments at the end of the period vary depending on whether the borrower defaults during the time step.
- The payments both at the beginning and end of the period vary depending on whether the borrower chooses to prepay.

Since the main objective is valuation, formulae are developed to express all these cash flows aggregated on a discounted basis at the beginning of the period. The cash flows for a simple bond and a default swap are described first, and then the more involved case of a complex credit facility.

Bond

Consider the simplest case of a bond. At each state and time step, some of the cash flows occur at the beginning of the period (in advance) and some occur at the end (in arrears).

The bond's cash flows are expressed as

$$CF_B = \begin{cases} AC + CF_{PP} & \text{if prepayment occurs} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$CF_E = \begin{cases} 0 & \text{if prepayment occurs} \\ CF_I + CF_P & \text{if no prepayment and no default occurs} \\ (1-L)(CF_I + AC) & \text{if no prepayment and default occurs} \end{cases} \tag{2}$$

where CF_B denotes cash flow at the beginning of the period; CF_E is the cash flow at the end of the period; AC is the commitment amount (which, for a bond, equals the principal outstanding); CF_{PP} is a prepayment penalty; CF_I is the cash interest payment; CF_P is the principal repayment owed and L is the loss severity rate.

Equations 1 and 2 show that if the borrower prepays, the holder of the security immediately receives the outstanding principal plus any applicable prepayment fee. Otherwise, the cash flow received at the end of the period depends on whether the borrower defaults during the time step. If the borrower does not default before interest and principal come due, the holder of the security receives the amounts owed in full at the end of the period over which those charges accrue. Alternatively, if the borrower defaults, the holder of the security receives only a portion $(1 - L)$ of the interest and principal owed. The timing of these cash-flow components is illustrated in Figure 1.

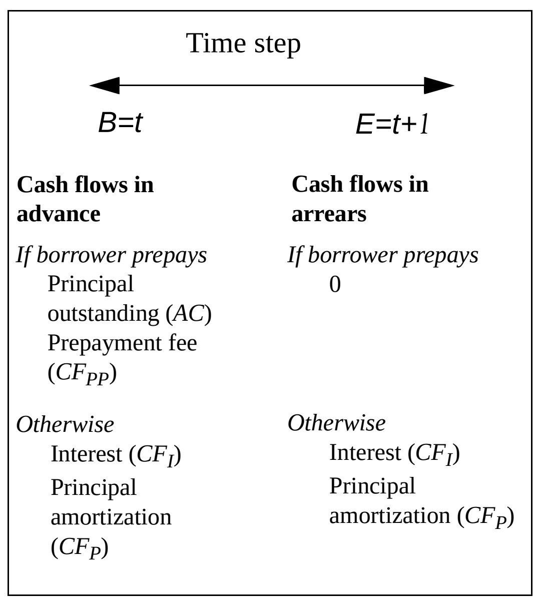


Figure 1: Usual timing of cash-flow components for a bond

This method of representing default proceeds is called the **recovery of par** or **legal claims approach** (see, for example, Duffie and Singleton 1999; Jarrow and Turnbull 2000). There are other conventional ways of modelling default losses. In the **recovery of treasury approach**, losses (or recoveries) are expressed as a fraction of the value of a risk-free bond (Jarrow and Turnbull 1995). In the **recovery of market value approach**, losses are expressed as a fraction of the value of the instrument just prior to default (Duffie and Singleton 1999). The remainder of this paper focuses on the legal claims approach.

For valuation, the cash flows at the beginning and end of the time step in Equations 1 and 2 can be combined on a discounted basis, using the discount rate known in the state at the beginning of the time step. The discounted cash flows at the beginning of the period are then given by

$$DCF = \begin{cases} AC + CF_{PP} & \text{if prepayment occurs} \\ (1+R)^{-1}(CF_I + CF_P) & \text{if no prepayment and no default occurs} \\ (1+R)^{-1}(1-L)(CF_I + AC) & \text{if no prepayment and default occurs} \end{cases} \qquad (3)$$

Here, DCF denotes discounted cash flow and R the applicable one-period (simple) discount rate, conditional on the state of the world at the beginning of the time step.

Assume that, at the beginning of the time step, default has not occurred and that, based on the time and state of the world, we know

- the risk-neutral prepayment probability, P_P
- the risk-neutral probability that default occurs during the time step, conditional on no prior default and all prior information, P_D.

Then, the risk-neutral expected value of cash flows discounted over the time step can be obtained by taking the expectation in Equation 3 with respect to the (one-period) risk-neutral default and prepayment probabilities to derive the expected discounted cash flow of a bond at the beginning of the period:

$$\begin{aligned} ECF &= (AC + CF_{PP}) \cdot P_P \\ &+ [(1+R)^{-1}((1-P_D)(CF_I + CF_P) \\ &+ P_D \cdot (1-L)(CF_{IS} + AC))] \cdot (1-P_P) \end{aligned} \qquad (4)$$

Equation 4 applies also to the risk-taking side of a total return swap with the bond as the underlying.

In the next two examples, we simplify the presentation by focusing only on expected discounted cash flows. In practice, all the conditional cash flows must be captured, without consolidation.

Credit-default swap

The one-period expected discounted cash flow of a credit-default swap is given by

$$\begin{aligned} ECF &= CF_{PP} \cdot P_P \\ &+ (CF_{DS} - CF_C - (1+R)^{-1} P_D \cdot L \cdot AC) \\ &\times (1-P_P) \end{aligned} \qquad (5)$$

Equation 5 can be understood as follows. A prepayment in this credit-default swap means that the protection buyer cancels the agreement. This event has a probability, P_P. In this case, the seller might receive a cancellation fee (CF_{PP}). Otherwise, if the contract continues, the buyer pays a premium at the start of the period (CF_{DS}) and the seller incurs servicing and monitoring costs (CF_C). If default occurs, the protection seller pays compensation ($L \cdot AC$) to the buyer at the end of the period, where AC is the committed amount.

Bank-credit facility

Bank-credit facilities sometimes allow the borrower to obtain credit by choosing from among a set instrument types. In the most general case, the borrower obtains credit by means of:

- a term loan
- a funded revolving line
- a letter of credit
- banker's acceptance.

Although it is rare for a single credit agreement to grant the borrower the option of choosing from among all of these instruments, the simultaneous use of all of these instruments leads to payments of interest and several different kinds of fees. The complexity of the resulting cash flows illustrates the required flexibility of the model. The timing of cash-flow components for a bank-credit facility is illustrated in Figure 2.

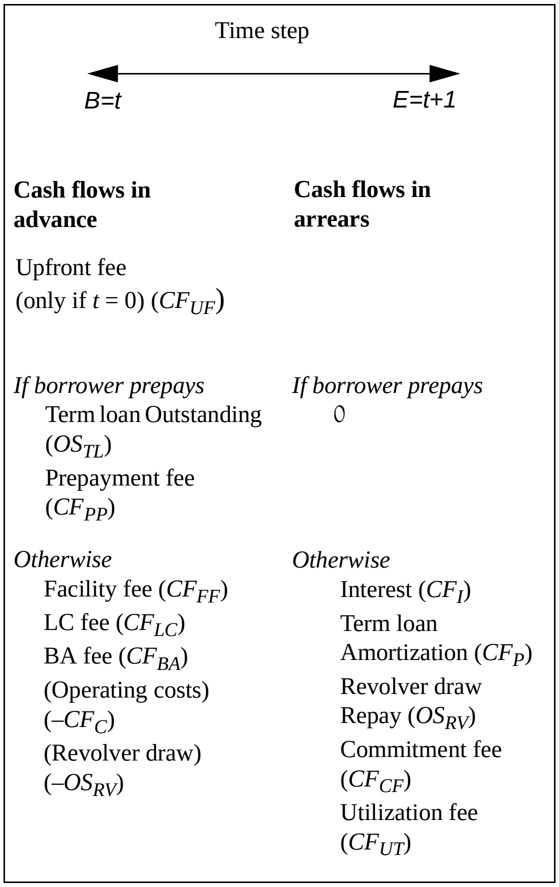


Figure 2: Usual timing of cash-flow components for a bank-credit facility

In bank-credit agreements other than straight, term loan facilities, the borrower has discretion, within limits, in choosing when to obtain credit, when to repay it and in what amounts. For modelling purposes, we assume that the borrower chooses the desired draw on a credit line at the beginning of each period and repays or cancels in full at the end of the period (as illustrated in Figure 3). This approach, in effect, treats the varying outstanding amounts in a credit line as a time series of differently sized one-period term loans. While this payment-and-draw pattern may not mirror the actual sequence of transactions, the state-contingent draws at the beginning of each time step offset any overstatement of repayment at the end of the preceding time step.

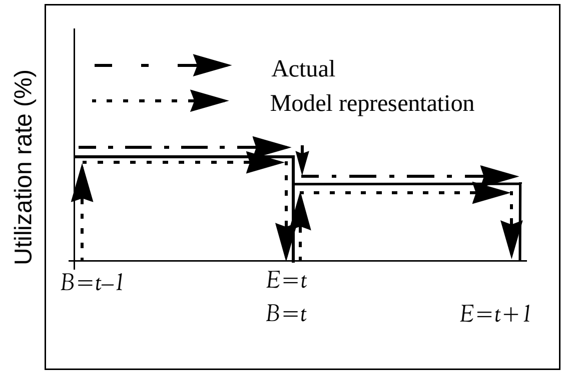


Figure 3: Modelling credit-line usage

Tables A1 to A5, in Appendix 2, summarize the relevant balances, bank cash flows, pricing rates, cost rates and utilization rates for a bank-credit facility.

The cash flows from a bank-credit facility include the following items paid at the beginning of the period:

- For a new facility ($t = 0$), the borrower may owe an "upfront" fee, CF_{UF}; at other times, $CF_{UF} = 0$.
- In the case of prepayment, the borrower returns the outstanding principal, OS_{TL}, and pays any applicable prepayment penalty, CF_{PP}. Thus, with probability P_P, prepayment occurs and leads to a total cash flow of

$$CF_{UF} + OS_{TL} + CF_{PP}$$

Note that, under this end-of-period revolver repayment convention, only the outstanding term loan amount is repaid at the beginning of the time step if prepayment occurs (see Figure 3). If no revolver draw occurs at the beginning of a period in which the borrower prepays, the repayment of the term loan reduces the outstanding balance to zero.

- If the credit facility continues, the borrower owes, at the start of the period, any applicable facility fees, CF_{FF}, letters of credit fees, CF_{LC}, and banker's acceptance fees, CF_{BA}. The borrower's draw of funds on a credit line, OS_{RV}, and the lender's expenses, CF_C, occur in advance. These items create cash outflows, which appear as negative entries. Thus, with probability $1 - P_P$, there is a total beginning-of-period cash flow of

$$CF_{UF} - CF_{FF} + CF_{LC} + CF_{BA} - OS_{RV} - CF_C$$

If the credit facility continues, several additional cash flows occur in arrears and the amounts realized depend on whether the borrower defaults:

- Interest, CF_I, commitment fees, CF_{CF}, utilization fees, CF_{UT}, and principal repayment, CF_P, come due at the end of a period. Also, by modelling convention, the funded revolving amount, OS_{RV}, is paid at the end of a period. Thus, in the absence of default, the total cash flow at the end of the period is

$$CF_I + CF_{CF} + CF_{UT} + CF_P + OS_{RV}$$

- In default, we assume that the borrower pays only the portion $(1 - L)$ of those amounts owed. The loss-in-event-of-default rate (L) reflects the seniority of the obligation, strength of covenant protection, the value and type of any collateral and the protection afforded by subordinated debt. Also, in default, the creditor receives only the portion $(1 - L)$ of the principal outstanding. Thus, all together, the cash flows at the end of the period if default occurs are

$$(1 - L)(CF_I + CF_{CF} + CF_{UT} + CF_P + OS_{RV})$$

- For credit lines with commitments available (i.e., when $AC > OS_{TL}$), the outstanding principal can rise as the borrower goes into default. The loan equivalency of the commitment, *LEQAC*, and the normal utilization rate, *REU*, determine the amount of this additional draw. Specifically, the funded outstanding amount in default is the sum of the normally drawn amount $(AC \times REU)$ and the normally undrawn amount, weighted by the *LEQAC* factor $(AC \cdot (1 - REU) \cdot LEQAC)$. The additional draw in default is then given by the expected outstanding amount in default, which is the sum of two terms

$$(AC \cdot REU + AC \cdot (1 - REU) \cdot LEQAC)$$

less the *funded* outstanding balance at the beginning of the period,

$$OS_{TL} + OS_{RV}$$

This contributes to an additional cash-flow loss at the end of the period

$$L[AC(REU + (1 - REU)LEQAC) - OS_{TL} - OS_{RV}]$$

This expression adjusts for the *additional* draw on a credit line that frequently happens as a borrower goes into default. For time steps as long as one year, this adjustment is needed to represent accurately the amount that will be outstanding and thus vulnerable to loss in default. For time steps as short as one month or one quarter, the *LEQAC* adjustment may be inappropriate.

Suppose that, during the year leading up to default, borrowers make additional draws of about 40% of the *original* commitment less the amount typically drawn; then, for an annual time step, $LEQAC = 40\%$. Assuming the normal utilization rate $REU = 30\%$ (which implies a normally undrawn fraction of $1 - REU = 70\%$), the expected usage in default is $0.30 + 0.70 \times 0.40 = 0.58$. The additional draw in default is thus $0.58 - OS_{TL} - OS_{RV}$.

The loan equivalency factor, *LEQAC*, measures the proportion of normally undrawn balances that have been drawn and thus are vulnerable to loss in the event of default. Thus, it reflects two competing effects:

- the deteriorating borrower's attempt to draw additional funds to cover an increasing cash-flow deficiency, and
- the lender's attempt to reduce the commitment available to a deteriorating borrower who predictably violates some loan covenants.

Weighting by the appropriate probabilities and discounting the cash flows occurring at the end of the period, all of these components are consolidated to obtain the expected discounted cash flow of the credit facility:

$$
\begin{aligned}
ECF = {} & (CF_{UF} + OS_{TL} + CF_{PP}) \cdot P_P \\
& + [(CF_{UF} + CF_{FF} + CF_{LC} + CF_{BA} - OS_{RV} - CF_C) \\
& + (1 + R)^{-1} \{(1 - P_D)(CF_I + CF_{CF} + CF_{UT} + CF_P + OS_{RV}) \\
& + P_D(1 - L)(CF_I + CF_{CF} + CF_{UT} + OS_{TL} + OS_{RV}) \\
& - P_D \cdot L \cdot (AC \cdot (REU + (1 - REU) \cdot LEQAC) - OS_{TL} - OS_{RV})\}] \\
& \times (1 - P_P)
\end{aligned} \tag{6}
$$

The *LEQAC* factor controls explicitly the usage of the credit line in default Equation 6. Moreover, it also controls the maximum usage of the credit line in non-default. Thus, it also affects several cash flows and outstanding amounts in Equation 6, through the credit line usage model. The *LEQAC* factor is further explained in the next section. Since one expects that the incentive to draw will be highest as the borrower goes into default, our assumptions do not allow usage in default to rise higher than that in a non-default situation.

Note that *LEQAC* measures the exposure in default as a fraction of the original, and not of the terminal, commitment. Its value can be imputed from market pricing of undrawn commitments or from past evidence on the usage of normally undrawn amounts in default. For example, suppose that market credit spreads on undrawn balances average about 25% of those on drawn balances. This motivates a *LEQAC* value of 25%. Alternatively, suppose that past data show that, in default, borrowers end up drawing about 50% of the commitment that was unused early in the life of the facility before any substantial decline in creditworthiness. This suggests *LEQAC* = 50%. Studies typically estimate *LEQAC* well below 100% and the Bank for International Settlements capital adequacy guidelines (BIS 1988) prescribes a value of 50% for undrawn commitments extended for one year or more.

The concept of a loan equivalency factor is familiar to practitioners exposed to BIS and internal capital allocation schemes. An alternative and more direct approach to using *LEQAC* is to model the credit line that the lender predictably achieves as the borrower's risk rating degrades. This can be seen as a lender's "option to reduce the line." Thereafter, the borrower is free to use the whole amount of the reduced commitment.

Several standard accounting relationships and other formulae ultimately tie the cash-flow components shown above to model inputs that describe the pricing and structure of the credit facility, market conditions and borrower behavior. Most of these primary relationships determine cash flows as the product of rates and balances. For example:

- The interest payable, CF_I, equals the product of the contractual interest rate, R_I, and the outstanding funded balance, using the proper day count and compounding conventions.
- The interest rate, R_I, equals either a specified fixed rate or the current value of the relevant floating rate computed as the sum of a base rate and a spread.
- In the case of a choice among varied floating rates, the option that provides the lowest rate, or the lowest rate that falls between an interest rate floor and ceiling, determines the floating rate.
- The spreads valid at the current time and state depend on the pricing grid, if there is one. Similar considerations arise in determining other cash-flow components.

To conclude this section, note that two different assumptions on operating costs may give rise to different expected cash flows and the value of the loan at each state and time. Operating costs can be seen from the perspectives of the market and the lender. The costs of efficient credit providers can be imputed from market spreads on loans. These market-derived costs affect market values, which, in turn, affect prepayment behaviour. Prepayment logically depends on the borrower's opportunities in the market as compared with the given loan. On the other hand, the lender's costs, as estimated possibly from activity-based studies, can differ from market-derived costs. In that case, the value of a loan from the viewpoint of the lender differs from its competitive market value. Assessing the difference between these two prices is an important exercise for the lender. We elaborate further on these two perspectives when discussing the prepayment modelling.

Modelling the embedded options

Equations 4 to 6 describe the expected discounted cash flows for a bond, a default swap and a general bank-credit facility, at a given point in time and state of the world. Embedded in these formulae are three types of options that depend on credit events:

With a **default option**, the borrower may not pay an obligation in full in the event of default. The expected cash flows of the bond, credit default swap and bank-credit facility are affected explicitly by this option through the probability of default, P_D, in Equations 4 to 6.

With a **prepayment option**, the obligor has the right to prepay commitments or cancel the contract at specified times before maturity. Prepayment generally depends on whether it is cheaper for the obligor to cancel the deal and enter into an identical one in the market, netting for cancellation costs and fees. This occurs when the market conditions (interest rates and spreads) move sufficiently in the obligor's favour or if there is a substantial improvement in creditworthiness, thus allowing the obligor to negotiate lower spreads.

The value of this option depends directly on both the market conditions and the creditworthiness of the obligor. In this sense, the option is contingent on credit events other than default (credit migrations). The expected cash flows in Equations 4 to 6 are explicitly contingent on prepayment through the probability of prepayment, P_P, which, in turn, is dependent on the credit state of the obligor as well as the level of risk-free interest rates and spreads.

With a **credit line utilization option**, an obligor has the right to choose the usage level of a given commitment. Of the three examples, only the bank-credit facility offers this option. It is generally the case that as an obligor's creditworthiness diminishes, the draw on the credit line increases. Therefore, as with the prepayment option, the credit line utilization contains an embedded option on credit events other than default, such as credit downgrades.

This option makes several terms in Equation 6 contingent on the state of the world, which now includes the credit state of the obligor as well as market conditions. The terms that are affected by utilization are the cash flows CF_{LC}, CF_{BA}, CF_C, CF_I, CF_{CF} and CF_{UT}, as well as the outstanding amounts OS_{RV}, OS_{LC} and OS_{BA}.

Default probabilities and credit migration are captured through the underlying credit model. The necessary characteristics of the credit risk model are described in the next section. In what follows, we describe the modelling of prepayment and line utilization, which occur simultaneously in a comprehensive framework. Hence, many of the same cost considerations apply in both cases.

Prepayment

It seems plausible to assume that the borrower will exercise the option to prepay a loan instrument if the market value of the loan, conditional on it continuing, *VNM*, rises high enough above par to pay for

- any prepayment penalty, given by a prepayment rate times the committed amount, $R_{PP} \cdot AC$
- refinancing transactions costs of the borrower, given by fixed and variable costs of searching for and negotiating a new loan, $FTC_{PP} + MTC_{PP} \cdot AC$
- origination costs, which are the (fixed and variable) costs that an efficient lender in the primary market incurs in originating a new facility, $FC_{ORIGM} + MC_{ORIGM} \cdot AC$.

Combining these three items, we obtain the total transaction cost of prepayment (TC_{PP}):

$$\begin{aligned} TC_{PP} &= R_{PP} \cdot AC + FTC_{PP} + MTC_{PP} \cdot AC \\ &\quad + FC_{ORIGM} + MC_{ORIGM} \cdot AC \end{aligned}$$

We assume that, in a given state of the world, the borrower will prepay if, in switching to a new loan with a competitive value of par in the secondary market, the savings relative to the existing above-par loan more than cover the transactions cost. Thus, the probability of prepayment in a state of the world, P_P, can assume only the values of zero or one and simply becomes an index of the prepayment event

$$P_P = \begin{cases} 1 & \text{if } VNM - OS_{TL} > TC_{PP} \\ 0 & \text{otherwise} \end{cases}$$

Although one could more generally model P_P as a continuous monotonic function of the predicted prepayment savings $(VNM - OS_{TL} - TC_{PP})$, in practice, it is difficult to obtain data to calibrate this function to actual borrower behaviour.

As an example, consider the workings of the prepayment model in the case of a $10 million facility. Suppose that as a result of an upgrade in creditworthiness, the facility's *NPV* in the market, conditional on no prepayment, rises to $150,000. Assume that, in refinancing the loan, an efficient lender will incur origination costs of $40,000 and that the borrower will incur search and negotiation costs of $15,000. Assume, further, that there is no prepayment fee. The total transaction cost of $55,000 falls short of the $150,000 gross savings that the borrower can realize from refinancing. The model will predict prepayment.

To implement this approach and ultimately determine the credit facility's value to a particular lender, both the lender's and the market's costs of originating and of servicing loans must be estimated. By "market" costs we mean those of competitive providers of credit. Borrower costs of transacting a new loan must also be determined. These estimates come from varied sources as described further in the data calibration section.

Credit line utilization

In bank-credit agreements other than straight, term-loan facilities, the borrower has the option to choose the usage of the line. Obviously, the line utilization is realized only in the event that the borrower does not prepay the facility. The usage of a line influences both the payments that the borrower owes to the creditor as well as the amount of exposure that the creditor bears. In Equation 6, the usage of the line affects several cash flows and outstanding amounts as described below.

The amount outstanding as a term loan, OS_{TL}, is fixed by the loan contract. Any remaining commitment above that amount is available to the borrower, assuming compliance with the loan covenants. The compliant borrower may use this amount in varying degrees from 0% to 100%. The usage model determines two components:

- the overall usage, *RUACA*, of the available commitment
- the relative usage of the different instrument options: the funded revolver, the letter of credit and the banker's acceptance.

The overall and relative utilization rates determine, in Equation 6, cash flows CF_{LC}, CF_{BA}, CF_C, CF_I, CF_{CF} and CF_{UT}, as well as the outstanding amounts OS_{RV}, OS_{LC} and OS_{BA}. The cash flows are obtained by multiplying contractual pricing rates by the corresponding drawn (outstanding) or undrawn (commitment less outstanding) balances. The outstanding amounts also influence operating costs and exposure.

We now describe the models for the overall and relative usage rates.

Overall usage rate, *RUACA*

The borrower's usage of the available commitment amount is modelled as a function of the net credit line cost. This can best be explained in several steps.

We start by defining the available commitment. Term loans basically involve a known schedule of outstanding amounts; hence, they are deterministic. All of the other bank loan types—funded revolvers, letters of credit, bankers' acceptances—involve outstanding balances that may fluctuate randomly. Thus, to determine the range of possible random variation in credit outstanding, we need to identify the commitment amount in excess of that set aside for a term loan.

The available commitment, *ACA*, given by the total commitment, *AC*, less any term-loan outstanding amount, OS_{TL}, can also be expressed as a proportion of the total commitment

$$ACA = AC - OS_{TL} = AC \cdot (1 - REUTL) \tag{7}$$

where *REUTL* denotes the portion of the facility devoted to a term loan. This is an attribute of the loan contract. In many cases, REUTL = 0%—a pure revolving line—or REUTL = 100%—a straight, term loan. Equation 7 also describes the case of a multi-instrument facility that includes a term loan as a component.

Then, the amount of available commitment that is outstanding, *OSACA*, is given by the product of the available commitment and its usage rate, *RUACA*:

$$OSACA = RUACA \cdot ACA \tag{8}$$

Usage rates vary widely depending on the purpose of a facility. A backup line generally has low usage rates, while an operating line has relatively high rates. The normal usage pattern is assumed to be provided by the user who knows the facility's purpose.

We assume that the borrower tries to minimize the costs of required credit. This suggests that the usage rate, *RUACA*, rises above its anticipated value if the marginal cost of drawing credit becomes cheap, and falls if the marginal cost becomes expensive. "Cheap" and "expensive" mean "low" and "high," respectively, relative to the market par cost of obtaining credit. If the cost of obtaining additional credit under the existing line rises far above the market par cost, then the borrower should be able to find cheaper credit elsewhere. The borrower would logically curtail usage of the credit line. If the cost under the existing line falls far below the market par cost, the borrower would tend to draw on the line and reduce the use of alternative credit.

Let CC represent the marginal cost of credit under the existing credit line and let MC denote the market cost, with both expressed in basis points. We define the net credit-line cost, *N* as

$$N = CC - MC$$

Appendix 3 explains how the marginal cost of credit, CC, and the market cost of credit, MC, can be computed in a given state of the world and time step.

As a practical matter, major changes in the net cost of drawing credit mostly reflect shifts in creditworthiness. One might expect the borrower to draw the maximum amount possible whenever the net cost falls below zero and draw nothing whenever it rises above zero. However, the available evidence generally suggests a less extreme reaction, where usage rises rather continuously as the credit rating degrades. Thus, a plausible model expresses usage as a logistic function of net credit-line cost, *N*, as shown in Figure 4.

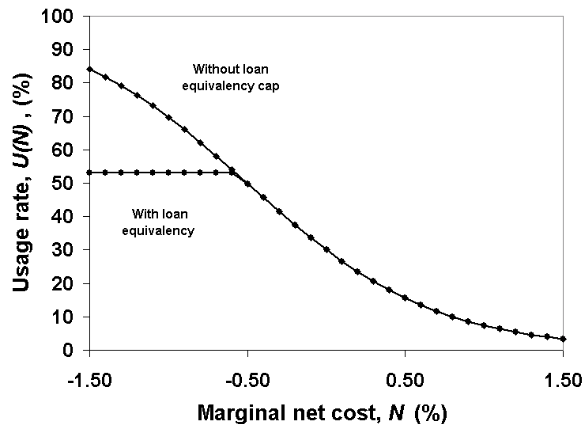


Figure 4: Capped logistic usage function

When the net cost is at zero, or at some other near par value established by assumption, the usage rate equals the anticipated value for the facility; that is, *RUACA*=*REU* (see Equation 6). As shown in Figure 4, when the net cost falls sufficiently below par, the logistic function is capped to reach a maximum, through the use of the loan equivalency of undrawn commitments (*LEQAC*). Usage reaches a minimum when the net cost rises well above par. The parameters of the logistic curve allow for calibration to the limited information on utilization patterns.

Recall that the *LEQAC* rate controls both the maximum usage in non-default and the usage in default. It represents, roughly, the effect of lender options to reduce a credit line as a borrower with deteriorating creditworthiness violates covenants.

If the contractual pricing of the facility does not itself depend on usage, then we obtain directly a single value for the net cost of a draw, *N*. The logistic function capped by the *LEQAC* factor in Figure 4 determines directly the usage, *RUACA*. Mathematically, we express this as

$$RUACA = U(N)$$

where $U(N)$ denotes the usage function in Figure 4.

However, some credit instruments include utilization fee schedules that assess incremental charges on usage in excess of specified thresholds. For example, a utilization fee schedule could specify no fee on utilization less than 33%, a 5bps surcharge on usage above 33% up to 50%, and an additional 5bps (for a total of 10bps) on incremental usage above 50%. In this case, a numerical algorithm must be used to solve simultaneously for the net cost, N, and the usage rate, *RUACA*.

Relative usage rates of different instruments

The usage model also deals with a general, multi-option facility in which the borrower has the right to draw credit as a funded balance, a letter of credit or a banker's acceptance. To model the relative usage of these different instrument options, one starts by determining which, if any, offers the cheapest cost of drawing credit. Then, relative to its anticipated value, the usage of any less economical instrument drops as its cost spread relative to the minimum cost option grows. Figure 5 gives an example of the relative usage function. The relative usage for a given instrument is given as a decreasing function of the spread gap to the minimum cost option.

This model of line usage accounts for the price of credit in contrast with approaches that tie usage only to the borrower's credit grade (CreditMetrics 1997). Such approaches imply that usage is the same at a risk grade regardless of the cost of obtaining credit and, hence, may not be very realistic.

Information on relative usage of alternative instrument options is very limited. The basic economic incentives faced by a borrower motivate the approach here. Parameters that limit the intensity of this economic incentive effect can be established if other considerations determine a borrower's relative usage of the different instrument options.

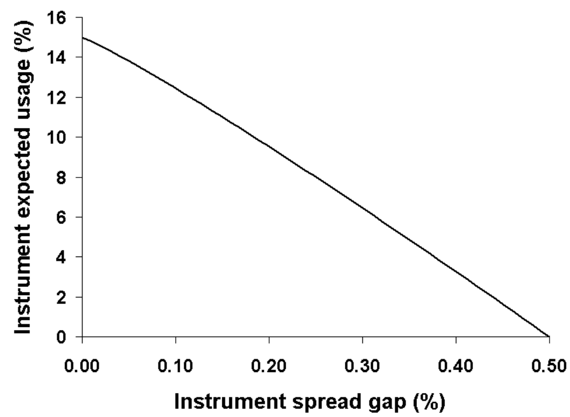


Figure 5: Example of relative usage schedule for instrument option

Credit risk valuation framework

As illustrated above, most credit agreements include key embedded options, notably the borrower's option to prepay or cancel a contract, and to draw on a credit line. As a result of these options, the cash flows from credit facilities vary with time, borrower creditworthiness (e.g., risk rating), interest rates and credit spreads. In particular, a decrease in interest rates or credit spreads or an improvement in borrower risk rating may trigger prepayment, drastically changing cash flows. Furthermore, more complex credit facilities also include additional features such as pricing grids, graduated utilization fees and amortization schedules that amplify the state and time dependency of cash flows.

In essence, we require an underlying credit risk model that describes each state of the world by

- the creditworthiness of the obligor (perhaps as given by a discrete set of credit ratings and default probabilities)
- the term structure of default-free interest rates
- the term structures of credit spreads for non-defaulted securities.

In principle, each of these sets of variables can vary stochastically through time. Intuitively, one would expect that at least three factors are required to capture this stochasticity in a reasonable way (e.g., a credit rating index, the risk-free short rate or forward rate and a systemic factor affecting credit spreads). The choice of the model, however, depends on trade-offs between the generality, complexity, speed, data requirements and accuracy of the model. For example, if one is interested in valuing floating rate instruments, then an assumption of deterministic risk-free interest rates and spreads may be appropriate and a one-factor model may be used. However, for fixed-rate instruments, it is important to have a stochastic model describing the evolution of risk-free interest rates. This can be done generally through a one-factor term-structure model. Although it may be tempting to use multi-factor models that better describe the evolution of the term structure, this may lead to an overall credit valuation model of dimensionality that is too high to be practical.

In what follows, some of the issues that influence the choice of the underlying credit risk model, the data required and the basic structure of the underlying pricing framework are highlighted.

Choice of underlying credit risk model

Credit risk pricing models are broadly classified in the literature into two main categories: the so-called structural approach and the reduced-form, or intensity-based, approach. See Das (1998), Duffie and Lando (1997) and Jarrow and Turnbull (2000) for comprehensive descriptions of the two approaches. (The reader is also referred to Aziz 1999a, 1999b, 2000) for some simple practical explanations of the general principles underlying these models.) Appendix 4 provides a brief review of these credit risk approaches.

In general, cash flows for loans vary with changes in the creditworthiness of a non-defaulting borrower, that is, movements between the various ratings grades short of default. Therefore, models that distinguish among many possible credit states, not just default and non-default, are required. Multi-credit state (rating-based) models seem particularly suitable for this problem (e.g., Jarrow, Lando and Turnbull 1997 (JLT); Lando 1998). In terms of applications specific to the loan market, rating-based models were first used in Ginzberg et al. (1994). Aguais et al. (1998) and Aguais and Santomero (1998) also describe valuation applications that use a multi-state model for evaluating the embedded options and other structural features found in loan instruments.

The potentially high dimensionality of rating-based models presents various theoretical and practical challenges. For example, in the most general case, a rating system with n credit states implies the need to calibrate on the order of n^2 parameters (rating transitions) per time step. Clearly, some reasonable structure to reduce the dimensionality of the problem must be added. The JLT model presents one such approach and Lando (1998) discusses more generally a number of additional practical approaches.

In general (assuming complete markets), no-arbitrage models require only pricing data for calibration. Rating-based models, in contrast, start by using real transition matrices (like those provided by S&P and Moody's) to describe the high-dimensional, discrete-transition probability space. One usually assumes that the evolution of the obligor's creditworthiness follows a time-homogeneous Markov process (in the real measure). A low-dimensional process is then applied to modify the transition matrix in order to fit to the observed term structure of market spreads. This yields the so-called risk-neutral measure. Typically, this calibration step may convert the process into a time-inhomogeneous Markov process (under the risk-neutral measure).

One can use the JLT or Lando low-dimensional process transformations to fit the observed credit spreads. However, in practice, this choice is not obvious. For example, since the JLT approach involves a proportional scaling of the transitions, it sometimes leads to numerical problems when applied in practice. Lando's approach using eigenvalue decomposition can also lead to practical numerical problems. Moreover, in both cases, the transformations are chosen for mathematical tractability and are not derived from underlying financial principles.

Note that by defining credit states as arising from an underlying process of the value of the firm, structural models can also be useful for this type of problem. However, these models may be difficult to fit to market data (particularly short-term spreads) and their use, in practice, may require some extra level of sophistication for modelling cash flows such as those found in loan instruments. A mixed model that combines functional parts of reduced form and structural models provides a sensible alternative.

The mixed approach assumes that there is an (unobserved, perhaps) underlying structural process, referred to as the creditworthiness index (CWI), determining a firm's credit state. This approach was first advanced in the CreditMetrics methodology (CreditMetrics 1997). As in the Merton model, default occurs when the CWI falls below a given threshold or default boundary. Also, one may define multiple thresholds that determine various credit states.

In the CreditMetrics approach, the default and migration thresholds are fit directly to match observed (one-year) rating migrations and default probabilities. Assuming that the underlying index is Gaussian, simple closed-form solutions can be obtained for one-period problems. As shown in Gordy (2000), Koyluoglu and Hickman (1998), and Belkin et al. (1998a,1998b), the model leads naturally to the modelling of stochastic default probabilities and transitions as a function of systemic risk factors. The CreditMetrics methodology solves the problem only for a single step. Multi-step versions of the methodology are presented in Iscoe et al. (1999) and Li (2000).

In summary, one may obtain a sensible underlying credit risk model for elaborating on the three factors outlined above:

- **Factor 1: borrower creditworthiness,** as designated by a set of discrete credit ratings
- **Factor 2: default-free short rate or continuous forward rate,** as determined by an HJM model **or discrete forward rate**, driven by a BGM model (At a higher computational cost, one may choose a higher dimensional model of the default-free term structure.)
- **Factor 3: systemic factor describing stochastic credit spreads,** from a stochastic intensity model or as the systemic component of a structural creditworthiness index.

Calibration data for the credit model and the cost models

The data required for calibrating the underlying credit risk model depends on its level of sophistication. For a model with deterministic spreads (two-factor model), the data for calibration include

- default-free term structure of interest rates
- defaultable term structure of interest rates (spreads) for each rating class derived from a combination of bond and loan data
- current transition matrix (as estimated, for example, from historical data as with ratings agencies or from a market-based model such as that of KMV or Moody's)
- recovery rates for each seniority and collateral class as well as by advance rate.

To include further stochastic spreads in the model, one also requires

- implied spread volatilities from traded instruments or, if these are not available, a time series of historical spreads or default/ transition data.

To obtain a correlated model of default-free and defaultable rates, one requires

- time series of default-free interest rates and spreads or, alternatively, observed default probabilities, to estimate correlations.

Ideally, we calibrate the model to prices of liquid, frequently traded credit instruments representing the economic regions, sectors and risk grades and terms needed for a comprehensive description of credit risk. It is important to note that one must extract option-adjusted spreads from the raw pricing information, to obtain the zero spreads used to calibrate the credit model. Recovery rates used in this calibration are usually drawn from bond and loan recovery studies (see, for example, Brand and Bahar (1998) and Eales and Bosworth (1998)). Finally, under the best circumstances, time series of credit prices to estimate spread are gathered.

Pricing data from both the bond and loan markets are used. Unlike bond spreads, loan spreads comprise an implicit cost component paid out of designated margins. As mentioned earlier, to determine the credit facility's value to a particular lender, one requires a model for both the lender's and the market's cost of originating and servicing loans. "Market" costs are those of competitive providers of credit. One also needs to estimate borrower costs of transacting a new loan. The estimates of lender, market and borrower costs come from varied sources.

For a particular lender, the cost information could derive from proprietary studies of credit activities at the institution. These costs might differ from market implied costs. While sometimes difficult in practice, market costs can be imputed from observed prices of loans and other instruments such as bonds. We illustrate this process with several examples:

- Suppose, in the secondary market, high-grade term loans have option-adjusted spreads that exceed those of comparable bonds by 45bps. That value might be taken as an estimate of the cost of servicing and monitoring those loans. In loans, the spreads pay for those costs, whereas, in bonds, they do not. Alternatively, if the servicing tranches on collateralized loan agreements offer spreads of about 40bps, that might be taken as a general estimate of servicing loans.
- With regard to origination costs, suppose that, following payment of reported upfront fees averaging 40bps, term loans in the secondary market trade at an average of 99.8% of par. We might conclude, therefore, that it costs lenders about 20bps to originate loans. Assuming that loans typically originate at par and solving for origination costs

$$100\% + FC_{ORIG} - CF_{UF} \times 0.4\% = 99.8\% \tag{9}$$

- Suppose that the combined selling and underwriting expense of bond issuance are reported to average 40bps. This can be used as an approximation of origination costs for syndicated loans.
- If almost no loan in the secondary market trades higher than 100.6% of par, an estimate of 60bps for the total of borrower transactions costs and lender origination costs might seem reasonable. Then, given a value of 35bps for competitive origination costs (in Equation 9), the borrower cost of searching for and negotiating a new loan is 25bps.

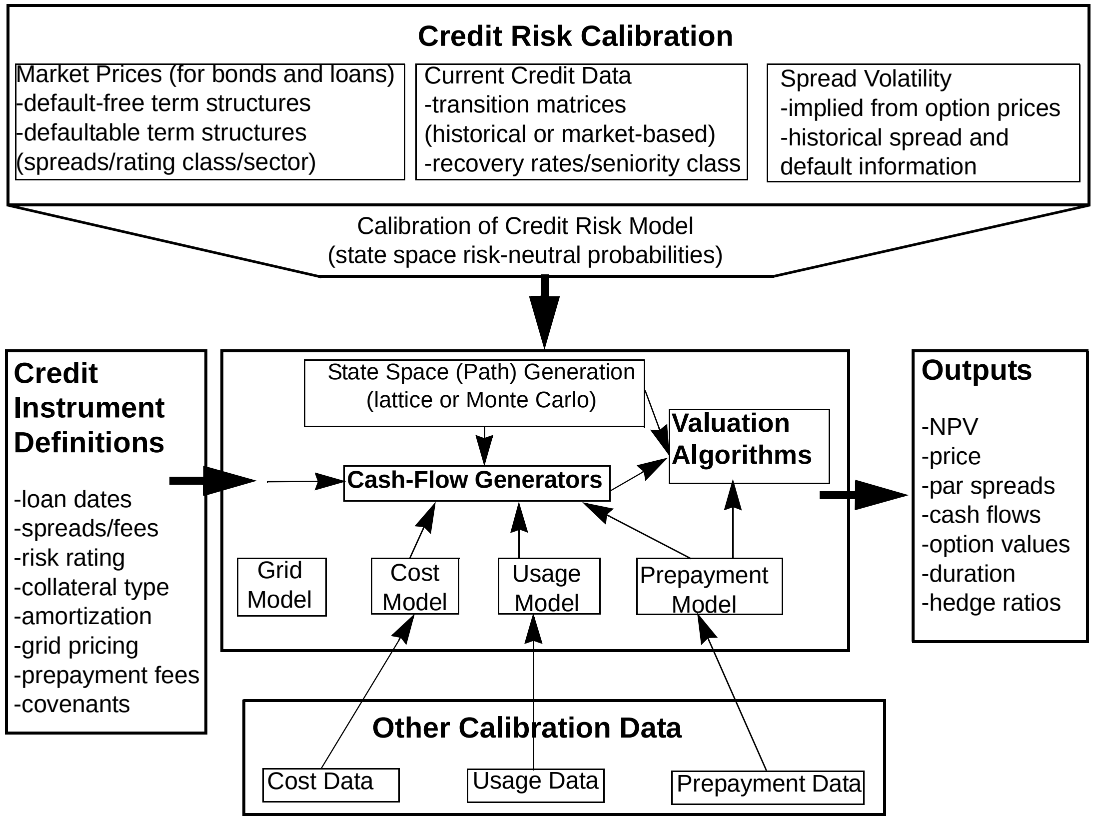
In most cases, these factors can be estimated only roughly, given the state of the current data. In particular, due to non-reporting of some upfront payments to lead arrangers, the available data could well understate lender origination costs. All such estimates of operating costs and of pure credit spreads (excluding costs) need, together, to reconcile with market pricing.

The loan market data cover mostly large syndications, with occasional sketchy reports on general trends in the US middle market. Data on investment-grade, syndicated loans come mostly from the primary market. Only speculative-grade loans trade enough for compilation of reasonably reliable secondary-market prices. Loan pricing vendors currently provide benchmark prices by credit grade for only two broad maturity bands—364 days and multi-year (four to six years on average). One vendor provides such pricing information for several industry groups. As suggested earlier, extracting the zero-rates term-structure (plain vanilla structure) from the raw prices may not be easy due to the complex structure and embedded optionality in loans. This process of adjustment depends on the model itself and can be complex.

Credit risk valuation architecture

We now describe the overall architecture required to support the pricing and structuring of loan instruments.

From a business perspective, the architecture must support the primary requirements of valuing each individual transaction at origination and MtM for an entire portfolio of credit instruments. The support of loan valuation over a set of future scenarios (Dembo et al. 2000) is also required for advanced portfolio credit risk solutions.



In terms of technology support for these two business objectives, valuation at origination requires pricing and structuring decision support for a large number of users in the front office, while MtM analysis typically requires middle-office, batch-mode analysis for an entire loan portfolio. Recent advances in technology, such as web-based tools, provide a platform for the deployment of this type of valuation framework, by placing decentralized valuation analysis much closer to the customer, while providing centralized management and control of complex analytics, key calibration parameters and credit data.

In Figure 6 we highlight the overall architecture needed to support implementation of this credit valuation approach, including five key components:

- **Credit risk calibration data**: this includes market data for bonds and loans (spreads), current credit data (default probabilities, transition matrices and recovery rates) and spread volatility data.
- **Usage, prepayment and cost calibration data**: this includes data to support the calibration of behavioural options components such as line utilization and prepayment. In addition, operating costs, which are part of the overall loan spread as highlighted, are determined by market-based information or a bank's own internal cost assessments.
- **Core analytics**: the core analytics include modules that determine option-exercise behavior and the generation of state-contingent cash flows. These are combined with the valuation algorithm, using either Monte Carlo or lattice-based methods.
- **Credit instrument definitions**: the specific terms and conditions of each credit instrument are inputs; the valuation outputs describe the loan's risk-adjusted characteristics.
- **Output reports**: these include prices, par spreads, cash flows, sensitivities, "what if" analyses and so on.

The pricing algorithm can be based on either a lattice-based or Monte Carlo-based approach. When working with up to three factors, lattice-based valuation methods are probably the most appropriate. In this case, a state-space lattice provides the framework that combines the cash-flow generating modules, which include the behavioral option-exercise logic, with the backward recursion algorithm that determines expected values. In particular, given that prepayment exercises depend on the actual value of the contract to the holder at a given state of the world and time, lattices provide a natural way to compute exercise boundaries (as is common, for example, with American options).

When the dimensionality of the model is higher, Monte Carlo methods are generally necessary to solve for the loan prices. These methods further allow for the handling of more complex path-dependent instruments, as well. However, Monte Carlo pricing usually has a high computational cost. Furthermore, as with American options, the practical implementation of the prepayment logic is more difficult in this case.

Concluding remarks

We present a general option-valuation framework for loans. While the focus is primarily on large, corporate and middle-market loans, the approach is applicable more generally to bonds and credit derivatives. This framework provides key valuation information during loan origination, and it supports MtM analysis for the entire loan portfolio, portfolio credit risk and asset and liability management. We emphasize the modelling of key product-specific features of loans and not the simple application of a specific type of pricing model to the problem. We describe the main structures found in commercial loans, such as utilization of credit lines and options to prepay, as well as outline the practical assumptions required to model the state-contingent cash flows resulting from these structures. The credit risk model characteristics necessary to capture the main features of the problem are also defined. Finally, we discuss briefly the families of credit models that may be appropriate for pricing, the data required for their estimation and reasonable criteria for choosing the sophistication of the model.

The proposed multi-state, ratings-based credit modelling approach with three factors captures the main characteristics of loans. By incorporating a stochastic interest rate factor, the model also values both floating-rate and fixed-rate credit instruments. Stochastic credit spreads can be supported by incorporating a systemic risk factor, which also captures the business cycle. This point is key since a prepayment-option exercise is driven by both movements between credit states and changes in the level of the term structure of credit risk.

As financial institutions progress toward applying MtM valuation to loans to support trading and credit risk transfer, this type of framework represents a key step forward. For those institutions that understand the arbitrage opportunities available in the loan market, the business benefits will be substantial. Implementation of valuation methods that incorporate detailed, state-contingent loan structures will also support improved estimates of portfolio credit Value-at-Risk.

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Appendix 1: Standard credit instruments

The simplest type of credit instrument is the (bullet) corporate **bond** (BD) in which the issuer obtains cash from the initial investors at origination and, in return, agrees to make payments of interest and, at maturity, of principal to holders of the securities. Some bonds include sinking fund or redemption provisions basically equivalent to amortization of principal. Most allow prepayment after a period of call protection. The bond's comparative simplicity makes it more readily marketable than other credit agreements that, in contrast, often include clauses proscribing or limiting assignments.

A **term loan** (TL) is a credit contract in which the borrower receives funds from the creditor(s) at contract closing or usually over a short period following closing and, in return, agrees to make payments of interest, fees and principal based on formulas and schedules specified in the agreement. Term loans can be quite complicated, involving amortization of principal, differing levels of seniority, posting of collateral, detailed covenant restrictions, prepayment penalties and interest and fees that may vary with the borrower's risk rating or financial performance. Term loans, however, account for a minority share of the lending by commercial banks.

A **revolver** (RV), or credit line or revolving line, is a credit agreement in which the borrower has the right to choose when to obtain funds and when to repay funds and how much to borrow, within limits set by the contract. These limits typically stipulate a maximum borrowing amount (commitment), the date by which all borrowed funds must be repaid, and the covenants that the borrower must satisfy to qualify for receiving funds. In some cases, the agreement requires that the borrower periodically "clean up" (pay down to a specified level) the facility before re-borrowing. The revolving line involves all of the complications of the term loan plus the added feature of granting the borrower the right to choose when to borrow and in what amounts.

Different types of revolving lines account for the major share of bank lending to businesses. By providing funds virtually on demand, the revolver allows a business to meet its working capital needs and to manage the liquidity risk created by volatile cash flows. By pooling revolvers across many businesses, a bank eliminates through diversification most of the liquidity risk that it inherits from customers.

Revolvers and term loans cover most of the lending that requires the creditor's money. Other standard types of credit instruments do not normally involve the bank or non-bank creditor actually lending money.

In a **financial letter of credit** (LC), the creditor guarantees the repayment of a counterparty's obligation and, in return, receives a one-time or periodic fee. Thus, a bank could issue a financial LC in support of a customer obtaining short-term cash from a money market fund that offers an attractive rate. In a financial LC, the bank essentially provides credit insurance. The instrument's contingent pay-offs mirror those of a credit default swap.

A **banker's acceptance** (BA) is another type of payment guarantee. In a BA, the bank certifies that it will stand behind time drafts (post-dated cheques) issued by a customer. The customer may then sell drafts endorsed as accepted by the bank at a discount to a funding source that does not want to bear the issuer's credit risk.

LC and BA facilities usually allow the borrower to choose when to make use of the credit support offered by the bank. Thus, the outstanding amount under these instruments may "revolve" in the same way as disbursed balances in a funded revolving line.

In a **credit-default swap** (DS) the buyer pays a one-time or periodic fee to the seller of protection for the right, in the case of default by a particular borrower, to receive cash compensation or to sell a credit instrument issued by the borrower at a specified price (near par). In contracts with extremely low-risk counterparties, this instrument offers basically the same state-contingent cash flows as a financial LC. Otherwise, the instrument involves counterparty risk as well as the risk of the underlying instrument. As with a financial LC and insurance contracts in general, the protection buyer in a DS typically has the right of cancelling (prepaying) the agreement.

In a **total-return swap** (RS) the protection buyer exchanges the total returns on a specified underlying debt instrument for a set of stable cash flows. The protection seller receives cash flows that match the interest and principal payments plus the gains (minus the losses) of the underlying instrument. As in the DS, the RS can involve counterparty risk in addition to the risk of the underlying instrument. Also, as in the DS, the RS usually allows the protection buyer to cancel the agreement.

We turn lastly to the most complex case, the **multi-option credit facility** (MOF). In an MOF, the borrower has access to a range of instrument types within a single facility or contract. In this case, the creditor commits to provide credit up to a maximum amount, which can amortize over time, to be drawn on in various ways largely at the borrower's discretion. In a more general case, the borrower can receive a term loan and then, as needed, obtain additional credit up to the remaining commitment amount. This additional credit can take the form of additional funded balances (revolvers), letters of credit, bankers' acceptances, or some combination of these types. Of course, an MOF can offer less than the full menu of instrument types.

Appendix 2: Summary of relevant cash flows

Variable	Description	Revolving (Y/N)	Derivation
AC	commitment amount	N	loan attribute from contract
OS	total outstanding amount	Y	$OS_{TL} + OS_{RV} + OS_{LC} + OS_{BA}$
OS_{TL}	term loan outstanding amount	N	$RU_{TL} \times AC$
OS_{RV}	revolver outstanding amount	Y	$RU_{RV} \times AC$
OS_{LC}	LC outstanding amount	Y	$RU_{LC} \times AC$
OS_{BA}	BA outstanding amount	Y	$RU_{BA} \times AC$

Table A1: Selected balances affecting bank loan cash flows and exposures

Variable name	Description	Timing (beginning or end of period)	Derivation
CF_{UF}	upfront fee	beginning	upfront fee rate × commitment amount
CF_{PP}	prepayment penalty	beginning	prepayment penalty rate × commitment amount
CF_{FF}	facility fee	beginning	facility fee rate × commitment amount
CF_{LC}	LC fee	beginning	LC fee rate × LC outstanding amount
CF_{BA}	BA fee	beginning	BA fee rate × BA outstanding amount
CF_{C}	operating costs	beginning	origination costs (t = 0 only) + servicing costs + collateral monitoring cost origination costs = fixed origination costs + marginal origination cost rate × commitment amount servicing costs = fixed servicing costs + marginal cost rate on outstanding × total outstanding amount + marginal cost rate on undrawn × (commitment amount – total outstanding amount) collateral monitoring cost = fixed collateral monitoring cost + marginal cost rate on collateralized outstanding × collateralized outstanding amount
CF_{I}	interest	end	contractual interest rate × (term loan outstanding amount + revolver outstanding amount)
CF_{CF}	commitment fee	end	commitment fee rate × (commitment amount – total outstanding amount)
CF_{UT}	utilization fee	end	total outstanding amount × blended utilization fee rate
CF_{P}	principal repaid (drawn)	end	term loan outstanding end of period – term loan outstanding beginning of period; determined by loan contract

Table A2: Selected bank loan cash-flow components

Variable	Description	Derivation
R_I	contractual interest rate	contractually specified fixed rate or minimum rate of floating rate options
R_{UF}	upfront fee rate	contractually specified
R_{CF}	commitment fee rate	contractually specified
R_{FF}	facility fee rate	contractually specified
R_{LC}	LC fee rate	contractually specified
R_{BA}	BA fee rate	contractually specified
R_{UT}	blended utilization fee rate	computed from contractually specified utilization fee schedule and current utilization as determined by usage model
R_{PP}	prepayment fee rate	contractually specified

Table A3: Selected pricing rates affecting bank loan cash flows

Variable	Description	Derivation
FC_{ORIG}	fixed cost of loan origination	estimated from pricing of small loans
MC_{ORIG}	marginal origination cost rate	imputed from secondary loan prices
FC_{SERV}	fixed cost of loan servicing	imputed from pricing of small loans
MC_{SERVOS}	marginal servicing cost rate on total outstanding amount	imputed from pricing of low-risk term loans
MC_{SERVAC}	marginal servicing cost rate on undrawn amount	imputed from undrawn pricing of low-risk loans
FC_{COLL}	fixed cost of collateral monitoring	imputed from pricing of small, secured loans
MC_{COLL}	marginal cost rate of collateral monitoring	imputed from default rates and pricing of secured and unsecured loans

Table A4: Selected cost rates affecting bank loan cash flows

Variable	Description	Derivation
RU_{TL}	term loan outstanding as percentage of commitment amount	loan attribute specified by contract
RU_{RV}	funded revolver outstanding as percentage of commitment amount	determined by usage model as influenced by the relative costs and anticipated usage rates of the different draw options
RU_{LC}	LC outstanding as percentage of commitment amount	determined by usage model as influenced by the relative costs and anticipated usage rates of the different draw options
RU_{BA}	BA outstanding as percentage of commitment amount	determined by usage model as influenced by the relative costs and anticipated usage rates of the different draw options
REU_{RV}	anticipated revolver outstanding as percentage of commitment amount	loan attribute entered by analyst
REU_{LC}	anticipated LC outstanding as percentage of commitment amount	loan attribute entered by analyst
REU_{BA}	anticipated BA outstanding as percentage of commitment amount	loan attribute entered by analyst

Table A5: Selected utilization rates affecting bank loan cash flows

Appendix 3: Marginal and market cost of credit

Let CC represent the marginal cost of credit under the existing credit line and let MC denote the market cost, both expressed in basis points. The net credit-line cost, *N*, is:

$$N = CC - MC$$

We compute MC by solving for the spread that implies an *NPV* of zero on a one-period term loan issued by the borrower

$$0 = \frac{(1+R+MC)(1-PD \cdot L)}{1+R} - 1 - MCOS$$

where, MCOS denotes the per-dollar cost of servicing and monitoring the term loan, as inferred from market pricing. Solving, we obtain

$$MC = (PD \cdot L + MCOS)\frac{1+R}{1-PD \cdot L} \qquad \text{(A1)}$$

Equation A1 shows that MC equals the sum of market-based credit and servicing costs, with some minor adjustments for payment timing and exposure to default.

The credit-line cost, CC, reflects the terms of the loan contract. Consider, for example, a revolver with a drawn spread over the risk-free discount rate of *RS*, a commitment fee of *RCF* and no other charges:

$$CC = RS - RCF$$

When the borrower draws, the interest-spread payments increase and commitment-fee payments simultaneously decrease. The marginal cost, CC, is computed by netting the two rates.

Suppose, alternatively, that the credit line is a *LC* facility, with a *LC* fee of *RLC* and a facility fee of *RFF*. Then,

$$CC = RLC\frac{1+R}{1-PD \cdot L}$$

The factor applied to the *RLC* adjusts for that fee being paid in advance rather than in arrears. The facility fee is not substituted, since, unlike commitment fee payments, facility fee payments do not decline with increasing line usage.

Suppose that $R = 6\%$, $RS = 175$bps, $RCF = 45$bps, $PD = 2.5\%$, $L = 30\%$, and $MCOS = 50$bps. Then,

$$CC = .0175 - .045 = .0130$$

$$MC = (0.025 \cdot 0.3 + 0.005)\frac{1 + 0.06}{1 - .025 \cdot 0.3} = .01335$$

$$N = CC - MC = -.00035$$

Therefore, $N = -3.5$bps, which indicates a small incentive to raise usage above initial expectations.

Appendix 4: Credit risk modelling approaches

There are two common approaches to credit risk modelling. The **structural approach**, originally developed by Merton (1974), treats the firm's asset-value process and its capital structure as the underlying determinants of expected default rates. Equity and debt of the firm are seen as options on the underlying firm's value. These models assume that default occurs if the firm's asset value falls sufficiently below the value of its debt. Instruments with credit risk to the firm are modelled as derivatives of the firm's asset value, and can therefore be priced using the Black-Scholes-Merton approach.

Early research in this area focused on developing explicit valuation formulas, given particular assumptions on the asset-value process and capital structure, and on comparing the values from those formulas with available market prices. More recent research attempts to explain features of market pricing by introducing jumps and informational imperfections into the valuation model (see, for example, Leland (1994); Longstaff and Swartz (1995a, 1995b); Duffie and Lando (1997); Madan and Unal (1998)).

The alternative **reduced-form** or **intensity-based approach** does not specify an underlying model of asset value and capital structure. Instead, default is modelled as an unpredictable jump event governed by a stochastic intensity process. The stochastic intensity processes describing default events or, more generally, credit-state transitions, are directly calibrated to market prices. By defining recovery rates in the event of default exogenously, no particular assumptions on the capital structure of the firm or priority in bankruptcy are required. Furthermore, given their mathematical tractability and similarities to terms structure models, the intensity models lead in an elegant way to Heath-Jarrow-Morton no-arbitrage conditions for defaultable debt (see, for example, Duffie and Singleton (1999); Madan and Unal (1998)).

The earliest reduced-form models deal with just two credit states (default and no-default) and make particular assumptions so as to obtain closed-form solutions for bond prices and facilitate model calibration to observed credit spreads (e.g., Jarrow and Turnbull (1995); Duffie and Singleton 1999)). Lando (1994), Jarrow, Lando and Turnbull (1997) (JLT) and Lando (1998) extend the reduced-form approach to the case of multiple, discrete, credit states or ratings. These models are sometimes referred to as **rating-based models**.

Under the JLT model, spreads are deterministic since both the migration probabilities and the recovery rates are also deterministic. Das and Tufano (1996) (DT) extend the JLT model to allow for stochastic credit spreads that may also exhibit correlation to the risk-free term structure. For mathematical simplicity, DT assume that transition probabilities are deterministic and that recoveries follow a mean-reverting process.

The intuition behind the DT model is that stochastic credit spreads arise when the underlying default probabilities (or the intensities) and/or the recovery rates are stochastic. Either one of these two conditions alone can be used to fit the model to observed spread volatilities. Instead, Lando (1998) and Jarrow and Turnbull (2000) use Cox processes to obtain stochastic intensities that also lead to stochastic spreads. Gaussian models generally are used only for tractability, although they may lead to negative intensities in practice. Alternatively, Duffie and Singleton (1999) propose modelling the intensities directly as a stochastic mean-reverting process (perhaps with jump terms).