# The effect of systematic credit risk on loan portfolio value-at-risk and loan pricing

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Using a contingent claims model framework, CreditMetrics derives correlations between ratings migrations of different borrowers from the observed correlations between equity values of the different industries and countries of those borrowers. As a variation of that analysis, we treat correlations here as arising from a single *systematic* risk factor. Applying this single-factor model, in Section I we evaluate the effect of systematic credit risk on loan portfolio value-at-risk. In Section II, we focus on individual loan transactions and assess the effect of systematic risk on credit spreads and, therefore, loan pricing and valuation decisions. Assessing this effect is critical, because arbitrage theory provides the basis for the Loan Analysis System℠ (LAS) developed by KPMG to support commercial loan valuation and pricing. Our results show that one must properly account for systematic factors separately from specific factors if one is to assess risk accurately at both the loan portfolio and loan transaction level.[1]

## I. Systematic credit risk at the loan portfolio level

In this section we evaluate the effect of systematic credit risk on loan portfolio value-at-risk. We compare the approach that considers only two credit states (default and no-default) with the one that distinguishes among multiple non-default states (the full-state model). In addition, we compare the portfolio payoff distribution under the full-state model with the corresponding Gaussian distribution that we would obtain if all borrowers were uncorrelated. Not surprisingly, we find that the Gaussian distribution poorly approximates actual value-at-risk. The two-state model provides a better estimate, though we still observe important discrepancies.

To model credit-risk correlation, we follow the CreditMetrics approach described by Gupton, Finger, and Bhatia (1997) and assume that transitions in discrete rating grades occur as a result of migration in an underlying process that measures “distance” from default. To simplify the presentation, we work with default distance transformed into a normalized risk score whose one-period changes have a unit normal distribution.

We index borrowers by $i$ and let $U_i$ denote the one-period change in the normalized risk score of borrower $i$. To represent correlation in a straightforward way, we assume that we can write each as:

$$[1] \qquad U_i = \sqrt{\rho_i}Z + \sqrt{1-\rho_i}\varepsilon_i$$

Here $Z$ represents a unit normal random variable measuring the composite effect of economic factors influencing the rating-score migration of all borrowers. We refer to this systematic component of credit risk as “$Z$-risk.” The $\varepsilon_i$ represent mutually independent unit normal variables (also independent of $Z$), each specific to an individual borrower. We refer to the credit risk induced by each $\varepsilon_i$ as “$\varepsilon$-risk.” The parameter $\rho_i$ determines the fraction of the variance of $U_i$ that is attributable to $Z$-risk. The correlation between $U_i$ and $U_j$ for any $i \neq j$ is $\sqrt{\rho_i \rho_j}$.

Since an investor can eliminate $\varepsilon$-risk through diversification, it commands no risk premium. $Z$-risk, on the other hand, appears in every portfolio of loans and in broad-based portfolios of other assets, no matter how varied. Since $Z$-risk cannot be diversified away, it accounts for the risk premium or “unexpected” credit loss charged on loans.

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[1] The methods we describe apply equally to loans, bonds, credit derivatives, and generally to any debt instrument whose cash flows are subject to a combination of interest rate risk and credit risk.

## The effect of systematic credit risk on loan portfolio value-at-risk and loan pricing *(continued)*

In what follows, we analyze the typical way in which Z-risk and credit correlation affect portfolio-wide risk. To simplify the analysis, we consider a loan portfolio in which each borrower has the same exposure to Z-risk. We thus replace the borrower-specific $\rho_i$ by a single value $\rho$ common to all borrowers.[2] The model in Eq. [1] then reduces to:

[2] $$U_i = \sqrt{\rho}Z + \sqrt{1-\rho}\varepsilon_i$$

The correlation between any pair of borrowers is now $\rho$. The two extreme cases are: (i) $\varepsilon$-risk only ($\rho = 0$) and (ii) Z-risk only ($\rho = 1$).

We define ratings by partitioning the $U_i$ into disjoint bins $[x_k^{(i)}, x_{k+1}^{(i)})$ defined by boundary values $x_k^{(i)}$. We fix the $x_k^{(i)}$ so that the one-period ratings transition probabilities calculated from Eq. [2] agree with observed historical rating migration probabilities. The method of calibrating the model found in Eq. [2] results in a separate set of ratings grade breakpoints for each starting grade.[3] This conflicts with the view that $U_i$ represents a simple transform of default distance. Also, by tying default to the position of $U_i$ at the end of the analysis period only, we depart from the view that default represents a trapping state in continuous time. Despite these limitations, the model has proved useful as a starting point in evaluating credit correlation.

Consider a two-period simple (option-free) term loan *L*. Let *X* denote the payoff to the lender at the end of the first period.[4] We apply the standard variance decomposition formula to *X*:

[3] $$Var[X] = E_Z[Var[X|Z]] + Var_Z[E[X|Z]]$$

The term $E_Z[Var[X|Z]]$ represents the average amount of payoff variability induced by $\varepsilon_i$. It measures (diversifiable) $\varepsilon$-risk. The term $Var_Z[E[X|Z]]$ measures the systematic variability in loan payoff induced by Z-risk.

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[2] The par credit spreads we refer to in what follows are those consistent with the market price of Z-risk. Therefore, the appropriate single value of $\rho$ in the present context is that associated with the well-diversified "market portfolio" of loans.

[3] That one does not obtain a consistent set of breakpoints across starting grades is a limitation of the assumed model for the underlying continuous risk rating process. The possible explanation is that the rating process is: (i) non-Gaussian, (ii) non-stationary, and/or (iii) not first-order Markov with respect to the current risk grade of the borrower. We note, for example, that empirical evidence is given in Carty and Fons (1993) that the recent past direction of movement in the borrower's rating (rating "momentum") can influence his subsequent rating behavior. This suggests that a second-order migration model which tracks both current risk grade and the rate (or at least direction) of change of risk grade may be appropriate.

[4] We think of the lender as selling the loan at the end of period 1 for its then fair market value. The cash flows constituting the payoff to the lender include the sale price of the loan plus any net income from the loan during period 1. We take as the fair market value of the loan at the end of period 1 its expected net present value under the risk-neutral (equivalent martingale) default/non-default transition measure. Use here is made of the fact (noted in Ginzburg, Maloney and Willner 1994) that one-period loans can be uniquely priced by arbitrage.

**The effect of systematic credit risk on loan portfolio value-at-risk and loan pricing** *(continued)*

Consider a loan portfolio $\wp_N$ that includes a large number $N$ of $L$-type loans made to different borrowers, each with the same initial rating.[5] Let $S_N$ denote the payoff of the portfolio $\wp_N$. The payoffs of the loans in $\wp_N$ are conditionally independent given Z. It follows, therefore, that

[4] $$E_Z[Var[S_N|Z]] = NE_Z[Var[X|Z]]$$

$$Var_Z[E[S_N|Z]] = N^2 Var_Z[E[X|Z]]$$

The systematic term in the variance decomposition of $S_N$ scales with $N^2$, while the diversifiable term scales with $N$. Thus, given positive correlation among the ratings migrations of different borrowers, for large enough $N$ the systematic risk will dominate in the variance decomposition

[5] $$Var[S_N] = E_Z[Var[S_N|Z]] + Var_Z[E[S_N|Z]]$$

For independent identically distributed variables, the central limit theorem applies to the partial sums $S_N$, with a normalization that scales with $\sqrt{N}$. In the present circumstance, we see that if any limit law holds, the normalization must scale with $N$. We now ask whether any counterpart to the central limit theorem holds.

To answer this question, we created a spreadsheet that calculates the probability distribution for the payoff from a portfolio of two-period loans. We set $N = 10,000$ and determined the distribution for the portfolio's net present value.[6] We also looked at the "normalized net present value," meaning the deviation from the portfolio's mean *NPV* divided by $\sigma_N = \sqrt{Var[S_N]}$. This represents the unexpected portfolio gain or loss measured in units of standard deviation. We use this normalized variable to quantify portfolio value-at-risk.

We analyzed a two-year loan under the following assumptions:

- A rating system with seven non-default rating grades (Aaa, Aa, A, Baa, Ba, B, and Caa);
- One-year rating grade transition matrix taken from Moody's Report (see Carty 1997);
- Constant risk-free interest rate = 4.5%;
- No loan origination or holding cost;
- Fixed loss in the event of default (*LIED*) = 40%;

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[5] The assumption that the loan portfolio is homogeneous to this extreme degree is made to simplify the analysis that follows. Qualitatively similar results would be obtained if the portfolio were heterogeneous with respect to initial borrower rating grade and loan term.

[6] We assume that $N$ = 10,000 is sufficiently large to very closely approximate the asymptotic limit distribution. Since the individual loan payoffs are conditionally independent given $Z$, the central limit theorem can be applied to the sum of the conditional loan payoffs. Asymptotically for large $N$ then, the portfolio payoff distribution is well-approximated as a weighted sum of Gaussian distributions, where the density for $Z$ determines the weights. The density corresponding to this weighted sum of Gaussians can be determined using numerical integration, so Monte Carlo simulation can be avoided. Our results show how highly non-Gaussian this weighted sum of Gaussians can be.

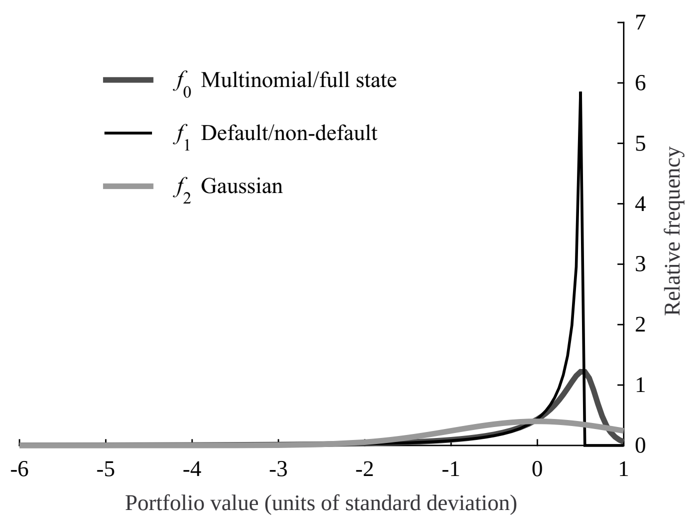
**The effect of systematic credit risk on loan portfolio value-at-risk and loan pricing** *(continued)*

- Credit risk premiums (unexpected loss) with a flat forward term structure: Aaa: 3.0 bps, Aa: 4.8 bps, A: 10.0 bps, Baa: 16.0 bps, Ba: 120.0 bps, B: 150.0 bps, Caa: 300.0 bps; and

- One of two values for credit migration correlation: $\rho = .15, .25$.[7]

We obtain a highly skewed value-at-risk distribution with long lower tail (Chart 1). We illustrate this by examining the limit density $f_0$ for the normalized portfolio value-at-risk in the case where (i) each borrower is rated Ba at loan origination, (ii) the correlation parameter ρ has the value .25, and (iii) each loan has a coupon of LIBOR + 171.9 bps (which represents par at origination).[8] In determining $f_0$, we translated the portfolio payoff by its mean value $E[S_N] = \$10{,}116.10$ and then scaled the result by the portfolio payoff standard deviation $\sigma_N = \$96.00$.

*Chart 1*
**Densities for loan portfolio value-at-risk**
*Initial borrower rating grade Ba*




For comparison, we have also displayed in Chart 1 the limit density $f_1$ that results if one assigns a two-period loan the value $1 (par value) at the end of period 1 if that loan does not default during period 1 and the value $(1 - LIED)$ if the loan defaults.[9] This collapses the migration process to two states: default and non-default. Under this default/no default model, the mean portfolio payoff is

[7] The values .15 and .25 are intended to be representative low and high credit migration correlations for companies with rated debt.

[8] The calculation of the par credit spreads is done separately for each initial rating grade. A trial-and-error search is performed until the expected net present value of the loan cash flows (including the disbursement and repayment of the loan principal) is zero under the risk-neutral rating migration measure.

[9] A loan which is originated at par but which is not grid priced can become significantly mispriced if the (non-default) rating grade of the borrower subsequently changes. However, absent the analytics to revalue the loan to account for the rating change (and possible further rating changes), a common accounting practice is for the lender to continue to carry the loan on his books at par.

## The effect of systematic credit risk on loan portfolio value-at-risk and loan pricing *(continued)*

$10,116.25, quite close to the value of $10,116.10 obtained under the full-state model. However, the portfolio payoff standard deviation is $83.26, substantially below the $96.00 value for the full-state model.[10]

Finally, we show the density $f_2$, the unit Gaussian density that would obtain for normalized value-at-risk if the migration processes of the borrowers were statistically independent. In that case, the central limit theorem would apply unconditionally and the density for the normalized portfolio payoff would closely resemble the unit normal.

Both $f_0$ and $f_1$ are markedly asymmetric and have modes that are skewed to the right. The mode of $f_1$ is more narrowly peaked than that of $f_0$. The upper tail of $f_0$ falls off smoothly, while that of $f_1$ is sharply truncated. The lower tails of the two densities differ in ways not discernible in the diagram. Consider, for example, the probability of a portfolio loss. The expected portfolio profit of $116.10 corresponds to $1.21\sigma_N$. Thus, portfolio losses occur if the payoff falls short of the mean by more than $1.21\sigma_N$. Under $f_0$ such an outcome occurs with probability .080. The corresponding probability under $f_1$ is .065. So if we were to use $f_1$ to approximate $f_0$, we would understate the probability of a portfolio loss.[11]

We of course observe a much more striking disparity between $f_0$ and the unit normal density $f_2$. The unit normal density ignores systematic risk, so it comes as no surprise that it poorly approximates $f_0$. The density $f_0$ is asymmetric, whereas the Gaussian density is symmetric. The density $f_0$ has a heavier lower tail. Under $f_2$, a portfolio loss occurs with probability .113. Thus, if we were to use the Gaussian as a proxy for $f_0$, we would overestimate the probability of a loss.

To summarize, our results show that if one doesn't distinguish among different non-default grades or especially if one doesn't account for systematic credit risk, one will make important errors in estimating portfolio value-at-risk.

[10] The density $f_1$ as plotted in Chart 1 (page 20) is based on the same location and scale change as that used to plot $f_0$, so that the two densities can be directly compared. In the particular case shown, it would have made essentially no difference whether the density was located relative to the default/non-default model mean or the full-state model mean. However, which scaling is used is important.

[11] In most loan agreements, the borrower has the right to prepay at par. If prepayment is allowed, then there would be less variability in the loan payoffs. The approximation to the multinomial payoffs provided by the two-state model would be closer than that shown in the text for the no prepayment case.

**The effect of systematic credit risk on loan portfolio value-at-risk and loan pricing** *(continued)*

## II. Systematic credit risk at the loan transaction level

In modeling credit risk, most analysts work with an ordered set of several non-default states as well as a single trapping default state. This multinomial framework creates a dilemma. One would like to apply arbitrage-free methods in pricing for credit risk.[12] However, in a finite (discrete time and discrete risk rating) model, these methods have found success in uniquely identifying prices only for binomial credit risks. "Binomial" refers to a model with only two credit states—default and non-default.[13]

Ginzburg, Maloney, and Willner (1994) resolve this dilemma by introducing one-period binomial reference loans with payoffs that approximate the one-period payoffs of the actual multinomial loan. Specifically, they construct binomial loans with payoff means and variances that match those of the one-period payoffs of the multinomial loan. For reasons that will be made clear below, we refer to this calibration as the *total risk method*. Each reference loan has only two possible payoff values. Thus, each one can be priced uniquely by arbitrage. The authors then outline a recursive procedure for computing the value of the actual multiperiod multinomial loan from the associated values of the binomial reference loans.[14]

Ginzburg, Maloney, and Willner (1994) consider a portfolio $\wp_N$ of $N$ statistically identical multinomial loans (each with the same multinomial payoff distribution) and a portfolio $\wp'_N$ of $N$ statistically identical binomial loans whose payoffs are those of the associated reference loans. They then argue that the value of portfolio $\wp_N$ and the value of portfolio $\wp'_N$ must approach equality as $N \to \infty$ . The argument relies on the central limit theorem.[15]

However, the needed assumption of statistical independence is problematic. If one could construct arbitrarily large portfolios of independent but statistically identical loans, the credit risks being priced would be fully diversifiable and would command no market premium. This conflicts with the observation that loan spreads in the market include a component for unexpected as well as expected

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[12] In applying the methods of arbitrage pricing, we make no claim that the actual secondary and syndicated loan markets are free of arbitrage opportunities. The loan markets lack the liquidity and product standardization of the equity markets, where the arbitrage-free assumption is a better approximation to reality. The justification for the use of arbitrage pricing is that it ensures that credit risks on more complex instruments are priced consistently with standardized credit risks and that the pricing methodology itself does not in effect introduce new arbitrage opportunities.

[13] In this case, the process splitting index is two, i.e., there are exactly two credit state values (default and non-default) which determine the loan payoffs. It is proved in Harrison and Pliska (1981) that a splitting index of two is the necessary and sufficient condition for unique arbitrage pricing to apply in the present finite model context. We observe that the analogue in the pricing of equities derivatives is that geometric Brownian motion is the asymptotic limit of a sequence of binomial random walks (each of which has a splitting index two).

[14] Based on this matching of the first two moments for each time $t$ and borrower rating grade $i$, a reference loan is constructed with payoff $HN(t,i)$ if non-default occurs, and with payoff $HD(t,i)$ if default occurs. The reference loan is valued under the risk-neutral measure for the default/non-default process. The recursion proceeds in reverse time from the loan end date back to origination. The expected loan NPV which results is taken to be the arbitrage value of the loan.

[15] Under the independence assumption, the payoff distributions of the two portfolios $\wp_N$ and $\wp'_N$ have the same mean and the same variance. Since the central limit theorem applies and both distributions are asymptotically normal, they must be asymptotically equal. If it is assumed that there are no "statistical" arbitrage opportunities (i.e., two credit risks with the same probability distribution of payoffs must have the same market value), then the common value of the loans in $\wp_N$ must be equal to the common value of the loans in $\wp'_N$. It is in this sense that the reference loan method in Ginzburg, Maloney, and Willner (1994) prices multi-period loan credit risks.

**The effect of systematic credit risk on loan portfolio value-at-risk and loan pricing** *(continued)*

credit loss. The market therefore indicates that the ratings migration processes of borrowers reflect systematic credit risk.[16]

This raises the question of how one should modify the calibration of the reference loan so as to account for systematic credit risk. The variance decomposition in Eq. [3] (page 18) provides the answer. We observe that only systematic risk commands a risk premium. Therefore, for two credit risks to be priced the same by the market, they should have the same amount of systematic risk. Thus the calibration in Ginzburg, Maloney, and Willner (1994) more properly would involve matching the systematic variance of the reference loan payoffs to the systematic variance of the multinomial loan payoffs. We refer to this scheme as the *systematic risk method*.

Alternatively, we could structure the binomial loan payoffs so that they create the least amount of systematic basis risk relative to the multinomial loan. In other words, we would structure the binomial loan to minimize the following expression:

[6] $$E_Z[E[X_B|Z] - E[X|Z]]^2$$

in which $X_B$ denotes the year-1 payoff to the binomial loan. This *basis risk minimization method* turns out to be mathematically equivalent to the systematic risk method if the conditional expectation $E[X_B|Z]$ of the binomial loan payoff and the conditional expectation $E[X|Z]$ of the multinomial loan payoff have a correlation coefficient of unity. For the relevant range of values of the problem parameters, we have observed this correlation to be .99 or higher. Furthermore, we have separately applied the systematic risk method and the basis risk minimization method and observed that the two schemes produce nearly identical loan values and par spreads.

We have created a spreadsheet that applies both the total risk method and the alternative systematic risk method. In this spreadsheet, we have represented the systematic risk variable $Z$ using 1,000 equiprobability bins. For each discrete value of $Z$ representing a bin, we determine the conditional moments for the payoff distribution both of an individual loan and a portfolio of 10,000 loans statistically identical to the given loan. In these calculations, we make strong use of the conditional independence of the portfolio loan payoffs for each value of $Z$.

We applied the total variance decomposition to the previously described two-period loan with the borrower initially in rating grade Ba, a (par) credit spread of 171.9 bps, and $\rho = .25$. The results indicate that, for an individual loan, most of the payoff variance (about 95%) reflects diversifiable

16 The Capital Asset Pricing Model (CAPM) relates the risk premium (return above the risk-free rate) on an investment to the correlation the return on that investment has with the "market" return. If a given investment has no correlation with the market, then again the risk is diversifiable and no risk premium is warranted.

**The effect of systematic credit risk on loan portfolio value-at-risk and loan pricing** *(continued)*

risk (Table 1). If we use the total risk calibration scheme in fashioning the binomial reference loan, we find that its payoff distribution understates the systematic variance of actual loan payoffs.[17]

At the portfolio level, virtually all of the payoff variance (99.7%) derives from systematic risk (Table 2). Furthermore, we now see a substantial gap between the payoff variances for the binomial- and multinomial-loan portfolios. Consequently, the second-order match enforced for individual transactions does not carry over to the portfolio.

*Table 1*
**Variance decomposition: individual loan**
*Values in USD*

| Statistic | Multinomial loan | Binomial loan |
|---|---|---|
| $E[X]$ | 1.011610 | 1.011610 |
| $E_Z[Var[X\|Z]]$ | 0.001750 | 0.001772 |
| $Var_Z[E[X\|Z]]$ | 0.000092 | 0.000069 |
| $Var[X]$ | 0.001842 | 0.001841 |
| $\sigma[X]$ | 0.042913 | 0.042912 |

*Table 2*
**Variance decomposition: portfolio of 10,000 loans**
*Values in USD*

| Statistic | Multinomial loan | Binomial loan |
|---|---|---|
| $E[S_N]$ | 10,116.10 | 10,116.10 |
| $E_Z[Var[S_n\|Z]]$ | 17.50 | 17.72 |
| $Var_Z[E[S_n\|Z]]$ | 9,197.68 | 6,913.57 |
| $Var[S_n]$ | 9,215.18 | 6,931.29 |
| $\sigma[S_n]$ | 96.00 | 83.25 |

These results show that the small systematic component of risk for individual loans almost entirely determines credit risk for large portfolios of loans. Risk decomposition therefore becomes an essential element in loan valuation and pricing.

To provide further insight, we examine the normalized portfolio payoff densities for loans with an initial risk grade of Ba and with credit migration correlations of $\rho = .15$ and $\rho = .25$, respectively. We first review the match between the multinomial and binomial reference loan portfolios for the

[17] We observed this to be the case for all initial rating grades and for varying values of the correlation parameter $\rho$. The conditional expectation $E[X|Z]$ of the loan payoff is inherently less variable as a function of the value of $Z$ in the binomial case than in the multinomial case. In the multinomial case, $E[X|Z]$ depends on all of the conditional rating migration probabilities $p(i \to j|Z = z)$. In the binomial case, the payoff depends only on whether default (D) or non-default (ND) occurs, so the only probabilities which matter are the conditional probabilities of default $p(i \to D|Z = z)$ and of non-default $p(i \to ND|Z = z)$. We have observed that changes in z can significantly affect the individual $p(i \to j|Z = z)$, while having relatively little impact on $p(i \to D|Z = z)$ and $p(i \to ND|Z = z)$.

case $\rho = .15$ (Charts 2a and 2b, on page 26) and then the case $\rho = .25$ (Charts 3a and 3b, on page 26). In each instance, we initially examine the accuracy of the reference loan approximations for the total risk method of calibration and then for the systematic risk method of calibration.

These comparisons are motivated by the principle advanced in Ginzburg, Maloney, and Willner (1994) that the closer the portfolios $\wp_N$ and $\wp'_N$ are in their payoff distributions, the closer the common price of the loans in $\wp_N$ should be to the common price of the loans in $\wp'_N$. We see that the agreement between the payoff distributions for the portfolios $\wp_N$ and $\wp'_N$ is closer (but still not exact) if the calibration is based on systematic risk as opposed to total risk.

We observe this improvement both when $\rho = .15$ and when $\rho = .25$. In each case, the locations of the density peaks become better aligned and the disparity in peak height diminishes somewhat. Using the systematic risk method, the variances of the portfolio payoffs for $\wp_N$ and $\wp'_N$ agree very closely. We get much less agreement when we use the total risk method. Finally, we obtain a closer match between the lower tails of the two densities using the systematic risk method, although that is not apparent at the scale at which the densities are plotted.

One concludes that credit risks in the finite model context cannot, in general, be priced exactly by arbitrage methods using binomial reference loans. Nonetheless, the accuracy of the reference loan technique in Ginzburg, Maloney, and Willner (1994) can be improved if the systematic risk calibration method is used in place of the total risk method.

To quantify the effect of the reference loan calibration method on loan pricing, we calculated the par credit spreads for the two-period loans under consideration. We determined these par spreads first using the total risk method and then using the systematic risk method (see Table 3 for the results when $\rho = .25$).

*Table 3*
**Effect of reference loan calibration method on par credit spreads**
$\rho = .25$

| **Method** | **Aaa** | **Aa** | **A** | **Baa** | **Ba** | **B** | **Caa** |
|---|---|---|---|---|---|---|---|
| Total Risk | 3.7 bps | 8.1 | 16.6 | 33.1 | 171.9 | 309.5 | 830.3 |
| Systematic Risk | 4.6 bps | 8.5 | 17.5 | 35.9 | 181.5 | 317.5 | 839.0 |

The par spreads are consistently higher under the systematic risk calibration method. This reflects our earlier observation that, using the total risk method, we underestimate the systematic component of loan payoff variance. Matching the systematic component of loan variance therefore causes the reference loan to appear more risky. This lowers the loan value to the lender and increases the required par credit spreads. The pricing differences are largest in relative terms at the higher rating grades and largest in absolute terms at the lower rating grades.

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**Loan portfolio value-at-risk densities**
*initial borrower rating grade Ba*

*Chart 2a*
**Total risk method,** ρ = .15

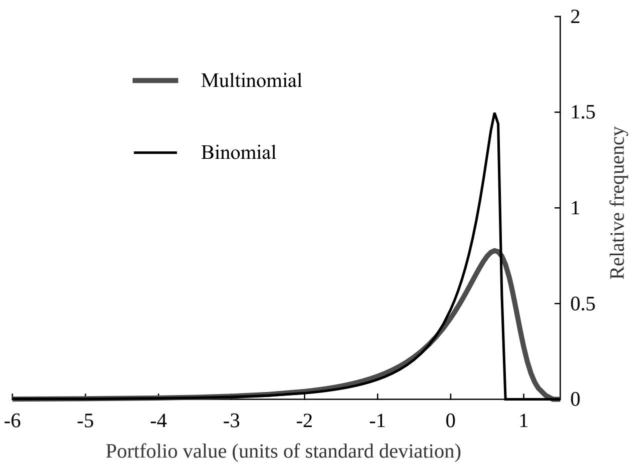



*Chart 2b*
**Systematic risk method** ρ = .15

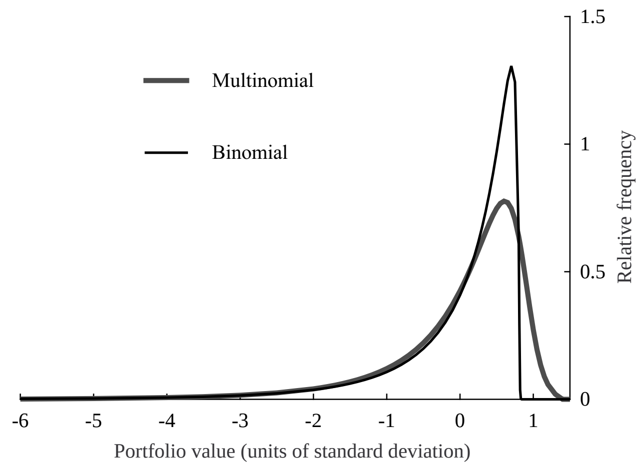



*Chart 3a*
**Total risk method** ρ = .25

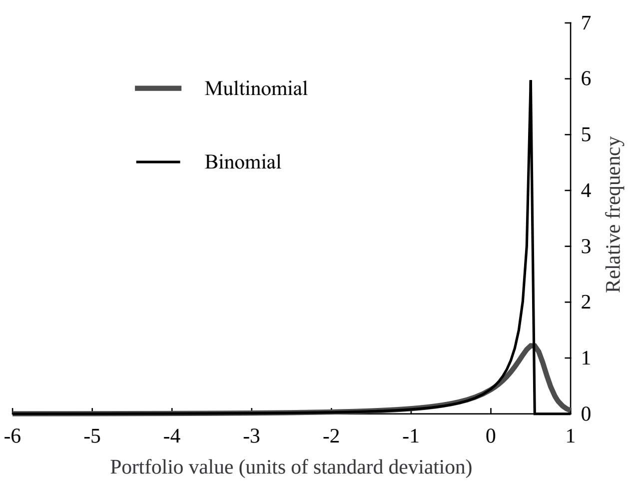
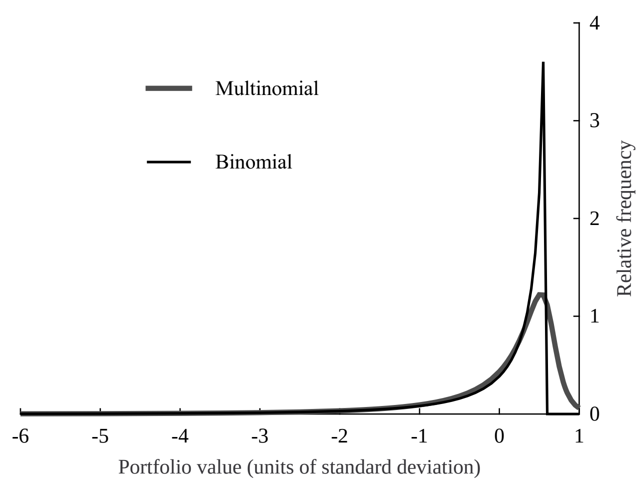



*Chart 3b*
**Systematic risk method** ρ = .25



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Incorporating the systematic risk method into the recursive valuation adds somewhat to the required computation. To implement the scheme we must:

1. Calculate the conditional one-period rating transition probabilities given each (discrete) level $z$ of $Z$-risk.

2. Determine the systematic component of variance $Var_Z[E[X|Z]]$ associated with the possible rating transitions out of each node $(t, i)$.

3. Calibrate the reference loan payoffs $HN(t, i)$ (the non-default payoff) and $HD(t, i)$ (the default payoff) at each node to match the mean and systematic variance of the binomial loan payoff to the mean and systematic variance of the corresponding multinomial loan payoff.

To perform these calculations, we must first estimate the correlation parameter. Under the CreditMetrics approach, one treats rating migration as driven by asset value movement relative to a default threshold. One may then estimate individual obligor correlations $\rho_i$ from industry and country asset correlations and the associated participation weights. One can then construct a well-diversified portfolio as a proxy for the market portfolio and define the market portfolio $\rho$ to be the weighted average of the appropriate $\rho_i$.[18]

In closing, we note that arbitrage theory provides the basis for the Loan Analysis System[SM] developed by KPMG to support commercial loan valuation and pricing. The desire to incorporate into LAS improved methods for the arbitrage pricing of credit risk motivated the work described in this article.

## Summary

Systematic credit risk accounts for the observation that credit risk cannot be "diversified away" even in large loan portfolios. To quantify the effects of systematic credit risk, we have postulated a one-factor model for credit rating migration that separates risk common to all borrowers from risk that is independent from borrower to borrower and therefore diversifiable.

Systematic risk causes the loan portfolio value-at-risk distribution to take on a distinctly non-Gaussian character. Relative to the unit Gaussian density, the mode of the normalized value-at-risk density is much shifted to the right and the lower tail is substantially elongated.

We also explored the effect of accounting for systematic risk but only distinguishing between default and non-default in credit rating migration. Our results indicate that this simplification also affects the overall shape of the value-at-risk density. The normalized value-at-risk density now has an exaggerated mode and a sharply truncated upper tail. This binomial approach also creates inaccuracy in determining the portfolio loss distribution quantiles.

[18] We have focused our attention on the case in which the binomial credit risk priced by the market and the unpriced credit risk specific to the obligor have the same $\rho$ value. However, the methods we have described apply equally to the case in which the unpriced risk has a fraction of systematic risk different from that of the priced risk to which it is calibrated. Once the price per unit of systematic risk is established, one need only determine how much of the related unpriced risk is systematic in order to price that related risk.

**The effect of systematic credit risk on loan portfolio value-at-risk and loan pricing** *(continued)*

Finally, we investigated the impact of systematic risk on arbitrage-free loan pricing. We observed that failure to distinguish between systematic and diversifiable risk in applying arbitrage methods creates pricing error. We then described the *systematic risk method* as an improved (but necessarily still approximate) technique for arbitrage pricing. It is characteristic of the systematic risk method that the value of a loan depends on the fraction of obligor credit risk that is systematic.

*The views and opinions are those of the authors and do not necessarily represent the views and opinions of KPMG Peat Marwick LLP.*

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