

Point-In-Time (PIT) LGD and EAD Models for IFRS9/CECL and Stress Testing

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ABSTRACT

In contrast with Basel-II rules, which call for the use of through-the-cycle (TTC) probabilities of default (PDs) and downturn (DT) loss-given-default rates (LGDs) and exposures at default (EADs), the regulatory stress tests and the new IFRS9 and proposed CECL accounting standards require institutions to use point-in-time (PIT) projections of PDs, LGDs, and EADs. By accounting for the current state of the credit cycle, PIT measures track closely the variations in default and loss rates over time. In past publications the authors have described the derivation of industry-region credit cycle indices (CCIs) and the use of those indices in converting legacy wholesale credit PD models, which typically understate cyclical variations, into fully PIT ones. This paper extends that framework to cover estimation of PIT LGDs and EADs for wholesale exposures. The authors offer options for the formulation of such models and discuss their experience in building PIT LGD and EAD models, and show that, by accounting for the probabilistic evolution over time in industry-region credit-cycle indices, one can derive joint, PD, LGD, EAD scenarios for use in the regulatory stress tests or in estimating the term structures of expected credit losses (ECLs) as needed for IFRS 9/CECL.

Keywords: Point-in-Time (PIT), Through-the-cycle (TTC), Loss Given Default (LGD), Exposure at Default (EAD), IFRS9/CECL, Expected Credit Loss (ECL), Stress Testing

1 OVERVIEW

The Basel II Advanced Internal Ratings Based (AIRB) approach have inspired financial institutions to develop models not only for PD, but also for LGD and EAD. In calculating RWA, the standards call for the use of TTC estimates of PD, which get converted to stress values via the IRB formula, and DT estimates of LGD and EAD. BCBS introduced the distinction between average and downturn LGD [1]. Paragraph 434 of [1] states that a higher than average DT LGD was a mechanism for capturing stressed conditions. Further, to this, paragraphs 468 and 475 in [1] specifies that institutions pursuing advanced approaches must use DT estimates of LGD and EAD under circumstances in which those measures exhibit cyclical volatility.

The use a TTC PD and fixed, DT settings for LGD and EAD helps stabilize RWA by making it insensitive to time varying credit cycle changes. This places the entire emphasis on the risk changes caused by an institution's portfolio choices and stops capital requirements from falling during a boom that could presage a decline, however, it also implies that RWA fails to measure current risk.

In regulatory stress tests and in new IFRS 9 and proposed CECL standards, the regulatory and accounting authorities have adopted the contrary position, asking institutions to apply their best, current-risk measures, the so-called PIT ones. For instance, paragraph 65 of the EBA's stress-testing-methodology document [2] states that, in all credit-risk-related calculations except RWA for all portfolios and not just AIRB ones, institutions should use PIT measures that reflect the current outlook for business-cycle conditions. The IFRS 9 requirements, paragraph 5.5.17, [3] state that ECL estimates should:

- correspond to unbiased, probability-weighted averages as determined by evaluating a range of possible outcomes, and
- draw on reasonable and supportable information that is available without undue cost or effort at the reporting date about past events, current conditions, and forecasts of future economic conditions.

We define unconditional PIT PD/LGD/EAD as an unbiased, unconditional estimate of default rate/loss/exposure over any specified horizon. Thus the term PIT measure is essentially a term structure as understood in literature and in this paper, we refer to models that produce the entire term structure. A good unconditional PIT estimate should account for all relevant information including the current state of the credit cycle and unconditional outlook for its evolution. We define PIT ECL in analogous fashion and believe that ECLs for IFRS9 and CECL should correspond to unconditional expectation of the future drawing on today's unconditional PIT measures.

We define conditional PIT PD/LGD/EAD as estimates of default rate/loss/exposure over any specified horizon, but which are derived based on occurrence of a particular macroeconomic or credit-factor scenario. A good conditional PIT estimate accounts for all relevant information including the current state of the credit cycle till today but only the specified macroeconomic or credit-factor scenario in the future. A good example is TTC measure which we think of as a special conditional case where today and future credit conditions are conditioned to be equal to long run credit conditions. We define conditional PIT ECL in analogous fashion and believe that ECLs for Stress Testing should correspond to conditional expectation of the future.

For clarity on terminology of PIT measures refer [10].

Ways for deriving PIT LGDs and EADs using the same general framework that has been previously used for PIT PDs is described in detail in the following sections.

2 PIT LGD MODEL MOTIVATION

A particular view of the default process influences the design of wholesale LGD models. Under that view, a firm defaults if its asset value falls below a threshold that implies a particular, asset-value deficiency relative to liabilities. The deficiency trigger could be 30%. If so, the LGD of all claims combined would be about 30%. But this is 30% plus or minus unpredictable variations attributable to the fuzziness of asset values in default and the haziness of the default-triggering mechanism. To recognize this LGD risk, one would account for the entire, LGD probability distribution function (PDF). Under circumstances with enough loss data to permit estimation of a PDF, several studies find that, in part due to threshold effects, the LGD PDF is distinctly non-normal. Indeed, one generally observes that, before accounting for recovery

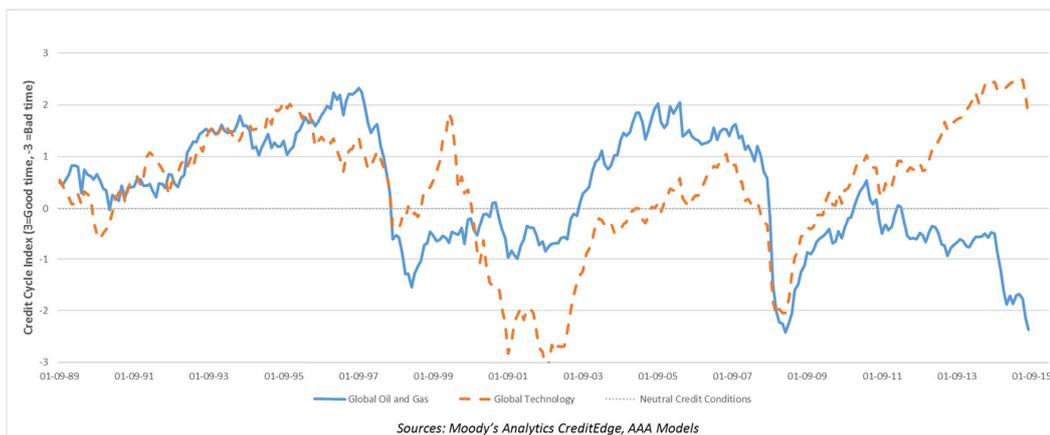
expenses, LGDs exhibit a bimodal distribution with point masses at 0% and 100% and a diffuse distribution between those extremes. Historically, over 40% of first-lien, US syndicated loans in default have experienced no loss. For loans to SMEs, the probabilities of both 0% and 100% LGDs can be substantial.

Among the factors with a predictable effect on the overall LGD, the credit cycle stands out as perhaps most important, since it implies that PDs and LGDs are correlated. Several studies present evidence of a relationship between the credit cycle and LGDs (see for example [4], [5] and [6]). A long list of references to earlier, LGD models in which aggregate DRs affects LGDs is presented in [6]. In terms of the conceptual view above, this credit-cycle effect indicates that the default threshold falls in downturns and rises in upturns. This in turn implies that, in downturns, defaults occur in smaller numbers but with larger LGDs than if the default threshold remained fixed. Despite this curious and generally unrecognized implication with respect to default incidence, the empirical relation between the credit cycle and LGD is well established and so has been embraced by the DT requirement of Basel II.

In previous research (see [7], [8], [9], [10], [11] and [12]) credit cycle indices have been used to translate PD models that understate the cycle into PIT ones. As a result, the translated PIT PDs best explain default rates. This study builds on previous research mentioned above and extends it to PIT LGD and PIT EAD and ultimately ECLs. Similar to their PD counterpart, the process of either converting a hybrid LGD model into a PIT LGD or re-developing a PIT LGD model is done using credit cycle indices.

These credit cycle indices are derived from summarizing, within selected industries and regions, PDs from a broad-based, fully PIT model such as Moody's CreditEdge. These indices are used as conditioning factors in models for deriving PIT LGD probability distribution function (PDF). The indices for distinct regions and industries allows one to recognize differences in the credit conditions of different sectors including the recent divergence between commodity producers and most other firms and between different countries/economies (see **Figure 1**). Industry effect is prominently highlighted in **Figure 1** as it is considered to be a strong driver in previous studies (see [13]).

Figure 1: Comparison of Credit Cycle Indices across different Industries



Among other potential conditioning factors, some studies find that the jurisdiction and industry of an exposure affect LGDs. Previous research by authors has indicated few, statistically significant industry effects other than for utilities, especially regulated ones. That defaults occur in a largely debt-financed business selling necessities priced at a mark-up over costs is hard to explain. It's not surprising, therefore,

that the LGDs are different, in fact much lower than in other businesses. Further, possibly reflecting the dwindling returns from recover efforts on ever smaller loans, some studies of SME LGDs find that very small loans have higher average LGDs.

Going beyond the overall LGD, the inputs with predictable effects on LGDs of individual claims mainly relate to priority. This explains seniority and security being key factors affecting LGD.

As a point of clarification, this section mostly refers to the modelling of so-called, ultimate-loss LGDs. One derives these LGDs from recovery information and not from debt-instrument, market prices shortly after default. One accomplishes this by: expressing the cumulative, recovery amount as a present, discounted value (PDV) as of the default date; converting that PDV into a percentage, recovery rate by dividing by EAD; and subtracting that recovery rate from 100%.

Ideally, one derives and models such LGDs for facilities. But institutions may sometimes consolidate the recovery information at the level of the legal entity. Under these circumstances, one obviously can't identify different outcomes for individual facilities.

See next a summary of some approaches to modelling wholesale LGDs as presented in several studies. Refer [4], [6], [13], [15], [16], [17], [18], [19] and [20].

3 PIT LGD MODELLING CHOICES

The choice of LGD modelling approach is largely dictated by the amount of data available for development and calibration. (see **Table 1**). For corporate entities, data sources are plentiful and this makes it possible to include more explanatory factors influencing outcomes For other portfolios options are limited by loss data. We made use of standard assumption of 9% discount rate.

Table 1 LGD model data availability for various portfolios

Portfolio	Data Availability to Large Institution	Possible Explanatory Factors
Corporates	4000+ loss observations from S&P/Moody's/GCD and perhaps 300 from the institution's experience over 10 years. Note that majority of the S&P/Moody's loss observations are North America based but there are more than a few hundred non-North American observations, enough to create representative robust models. Moreover, GCD is European focussed and has robust coverage.	<ul style="list-style-type: none"> • Seniority <ul style="list-style-type: none"> – Ordinal: Senior Secured, Senior Unsecured, Senior Sub, Sub, Junior – Cardinal: Debt Senior (%), Debt Junior (%) • Security <ul style="list-style-type: none"> – Ordinal: First Lien, Second Lien, ..., Unsecured – Cardinal: Collateral Coverage (%) • Facility type: Revolving Credit Facility, Term Loan, Bond; proxy for 'essentialness' • Resolution time: proxy for asset-value deficiency (default point) • Size: important for smaller facilities; proxy for 'recovery effort' • Utility dummy • Industry-region specific Credit Cycle Index
Banks	>100 S&P/Moody's observations and about 50 from an institution's experience over 10 years	Seniority, Advanced/Developing Economy dummy, Security, Industry=Banking and Region Specific Credit Cycle Index. Separate treatment for Trade Finance and Covered Bond facilities

Portfolio	Data Availability to Large Institution	Possible Explanatory Factors
Property	>1000 observations from a vendor such as Trepp and perhaps 200 from an institution's experience over 10 years	Property type (e.g. residential, office, retail, industrial, hotel, healthcare), Credit Cycle Index.
Project Finance	>200 observations from S&P/Moody's Project Finance Consortium and perhaps 30 from institution's own experience over 10 years	Asset Type, PFI/PPP and Gulf/non-Gulf country dummy variable, Industry-region specific Credit Cycle Index

One also needs to consider the specification of the model that links explanatory variables to LGDs. Various functional forms for LGD model specification are described in [21]. Once again, data availability affects the choices available. Options generally considered for various wholesale credit portfolios are presented in **Table 2**.

It makes sense to choose the best approach allowed by the data. Collecting loss data is difficult and costly. For those with the necessary expertise, estimating and implementing LGD models is comparatively easy and cheap. Thus one has difficulty justifying inferior approaches. If ample data are available, statistical best practice calls for modelling the entire LGD PDF, with the central tendency and spread of realized values influenced by factors such as seniority, security, and the credit cycle. One needs PDF approaches for hypothesis testing and objective, model validation. Specifications for an LGD PDF include:

- Tobit
- Zero-One-Inflated-Beta

If, however, data are sparse then one may need to settle on a simpler model, with perhaps the simplest form least demanding of the data being a look-up function determining an expected value for each of a number of categories defined by such things as seniority, security, obligor/facility type, and downturn/non-downturn.

Table 2: Various PIT LGD model specification options

Model Type	Model Description	Model Output	Pros	Cons	Typical Portfolios
Look Up Table	Model inputs determine cell in the look-up table	Point Estimate	Feasible with small samples	Cliff effects; no hypotheses tests	Funds, Sovereigns, Insurance Companies
Parametric Expected-Value Model	Model inputs enter into a formula providing a point estimate	Point Estimate	More accurate than look-up; no cliff effects	Calibration requires comparatively large samples; no hypothesis tests	Corporate entities.
Tobit	Model inputs determine mean and standard deviation of a latent normal distribution	PDF	More accurate than look-up; no cliff effects; facilitates hypothesis tests	Calibration requires comparatively large samples; unimodal form may be counterfactual	Corporate entities, Banks, Property
Zero-One Inflated Beta	Model inputs determine 0 and 1 probabilities and alpha and beta	PDF	More accurate than look-up; no cliff effects; facilitates hypothesis tests;	Calibration requires comparatively large samples; more complex than Tobit	

Model Type	Model Description	Model Output	Pros	Cons	Typical Portfolios
	parameters of beta distribution		more flexible than Tobit		

One may formulate a PIT LGD model using a Tobit specification is as follows

$$\begin{aligned}
 P(LGD_i = 0) &= \Phi\left(-\frac{m_i}{s_i}\right) \\
 P(LGD_i = 1) &= \Phi\left(-\frac{1-m_i}{s_i}\right) \\
 f(LGD_i | 0 < LGD_i < 1) &= \frac{1}{s_i} \Phi\left(\frac{m_i}{s_i}\right) \\
 m_i &= m_0 + \sum_k m_k \cdot x_{k,i} + m_Z \cdot Z_{IR(i),t} \\
 s_i &= \exp\left(s_0 + \sum_k s_k \cdot x_{k,i} + s_Z \cdot Z_{IR(i),t}\right)
 \end{aligned} \tag{1}$$

In which $P(LGD_i=0)$ and $P(LGD_i=1)$ are the probabilities of 0% and 100% LGDs for the i^{th} case
 $f()$ is the LGD probability density function for $0 < LGD < 1$ for the i^{th} case
 Φ - standard normal cumulative distribution function
 m is the latent mean for the underlying normal function
 s is the latent standard deviation for the underlying normal function
 m_k – regression coefficient for k^{th} factor in latent mean
 s_k – regression coefficient for k^{th} factor in standard deviation
 $x_{k,i}$ – k^{th} independent factor like %Debt Above, %Debt Below, etc for the i^{th} case
 m_0 – intercept component of latent mean
 s_0 – intercept component of latent standard deviation
 m_Z – intercept for credit cycle index component in latent mean
 s_Z – intercept for credit cycle index component in latent standard deviation
 $Z_{IR(i),t}$ – Industry Region specific credit cycle index for i^{th} case at time t

The Tobit provides perhaps the simplest, PDF model that explains point masses at 0% and 100% and a smooth distribution between those extremes. It involves estimation of two parameters, a mean, m , and a standard deviation, s , both of which may be functions of seniority, security, the credit cycle, and other factors. One calibrates the model using maximum likelihood estimation (MLE).

$$\underset{\{m0, mk, s0, sk, sZ, sZ\}}{\text{Max}} \sum_i (\delta_{i0} \ln(P_i(0)) + \delta_{i1} \ln(P_i(1)) + (1 - \delta_{i0} - \delta_{i1}) \ln(f(LGD_i)))$$

$$\delta_{i0} = \begin{cases} 1 & \text{if } LGD_i = 0 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

$$\delta_{i1} = \begin{cases} 1 & \text{if } LGD_i = 1 \\ 0 & \text{otherwise} \end{cases}$$

$LGD_i = LGD$ for i th observation

In this formulation, the factor $Z_{IR(t)}$ stands out as key, since it ensures that the model is PIT. The remainder of the formulation explains cross-facility and cross-default-case, idiosyncratic variations. Note that, unlike in the modelling of PDs, under the current, state-of-the-art one assumes that idiosyncratic effects occur only at the default time, not in a cumulative fashion over time. This reflects an assumption that the inputs into LGD models other than Z factors remain static, unchanging in response to random variations over time in the related entity’s creditworthiness. This may not be entirely true, but so far no one has produced data allowing one to test for “structural migration.” Thus, only the Z factor explains inter-temporal variations in LGDs. One can identify the significance of this effect through a simple likelihood ratio test of the credit-cycle index’s contribution as an additional factor.

In the Tobit model above with the parameters set to values like those obtained in past work, a decrease in Z from 0 to -2, which represents a two annual-standard-deviation improvement (deterioration) in credit conditions, causes the expected value of the LGD of a typical loan to rise by roughly 10 percentage points. An increase in Z from 0 to 2 implies an LGD decline of about the same magnitude. Such sensitivity forms the basis for a PIT LGD model. Implemented in a batch process, an institution could use the model in updating regularly its PIT LGDs to reflect current and projected, credit conditions.

The manner in which changes in the relevant, Z factor value from 0 to +/- 2 affect the illustrative Tobit PDF is illustrated in **Figure 2**, **Figure 3**, and **Figure 4**. The figures provide a visual feel of how the PDF shifts with change in Z factors. One observes that, as the Z factor turns increasingly negative, the continuous part of the distribution shifts up (toward higher LGDs) and the probability of a 0% LGD falls and the probability of a 100% LGD rises. One sees the opposite pattern as Z turns increasingly positive. The expected values of LGD in these cases if Z being at -2, 0, and 2, respectively, are 36%, 25% and 14%.

Figure 2: One Year PIT LGD Tobit Distribution for Z = -2

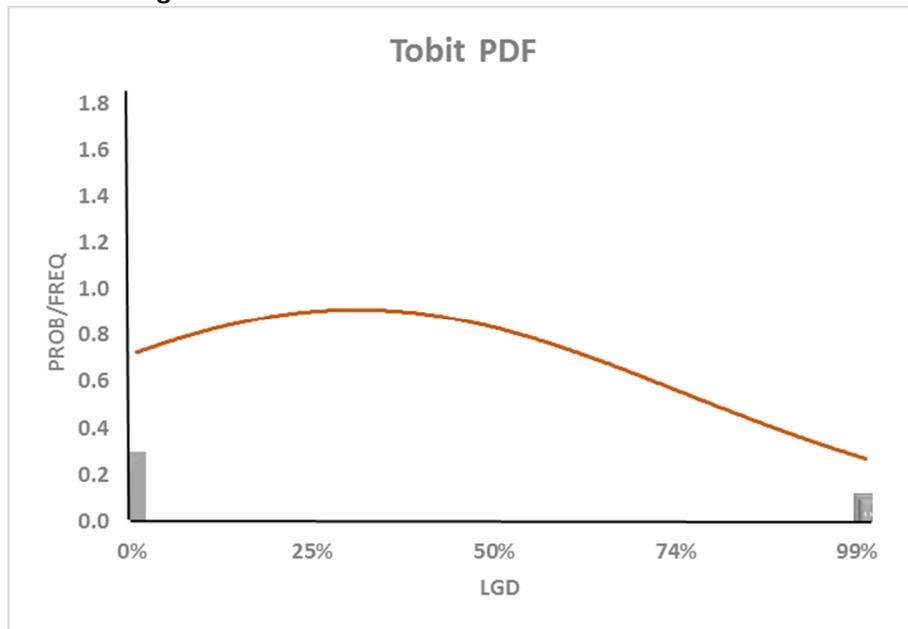


Figure 3: One Year PIT LGD Tobit Distribution for Z = 0

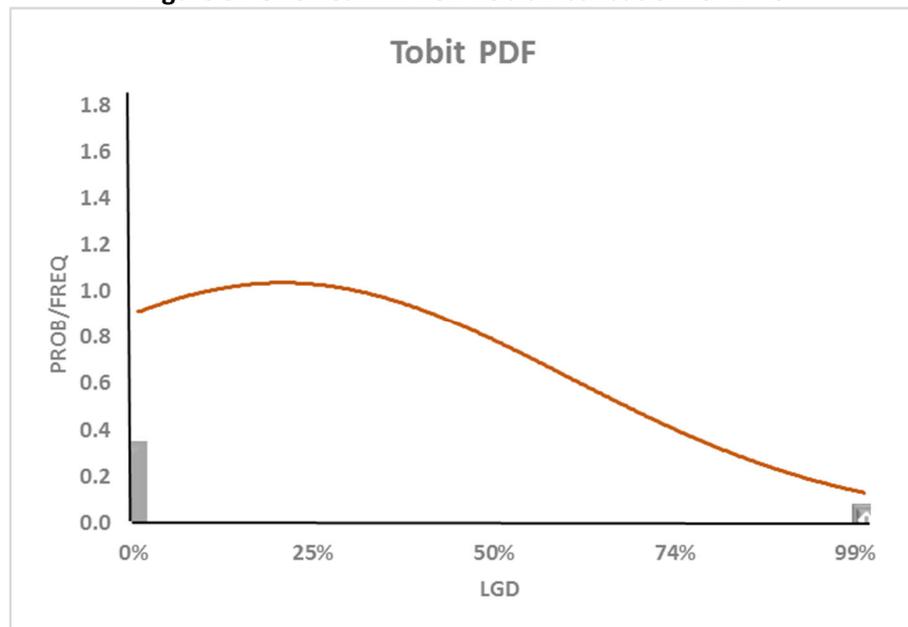
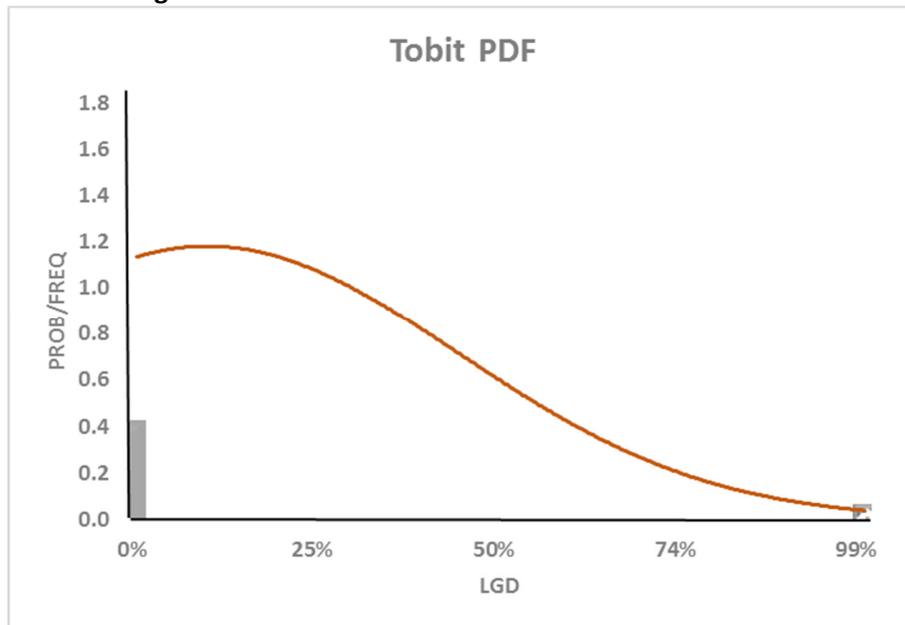
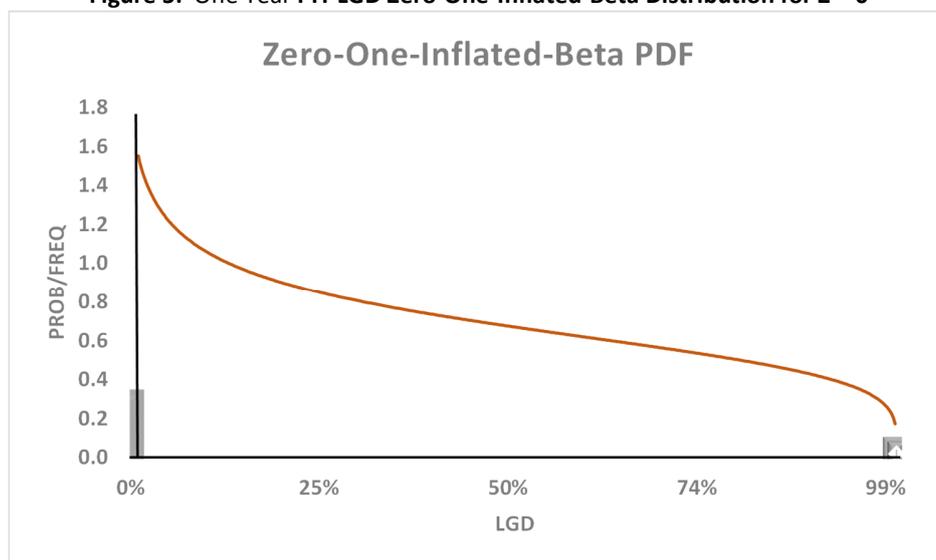


Figure 4: One Year PIT LGD Tobit Distribution for $Z = +2$



One can similarly derive a PIT LGD models based on the Zero-One-Inflated-Beta specification. This specification provides for a more flexible description of the data at the expense of more parameters to estimate. Unlike the Tobit, the continuous part of the distribution doesn't necessarily involve a central mode and this sometimes helps in fitting to SME outcomes with appreciable probabilities at both 0% and 100% (Figure 5).

Figure 5: One Year PIT LGD Zero-One-Inflated-Beta Distribution for $Z = 0$



Most applications call for the use of expected values of LGDs, often expectations conditional on particular Z scenarios. The models providing point estimates produce only these expected values. In the case of the PDF models, one can easily compute them from the PDFs.

4 PIT EAD MODEL MOTIVATION AND OPTIONS

The models that explain EADs under credit lines have the following motivation. Firms under duress draw down their lines so as to pay bills and buy time to turn the business around and avert default. This simple logic implies that EAD depends on the amount of credit outstanding and the amount of unused headroom. Of course, institutions providing credit lines to a firm in distress have incentives to curtail access to those lines. Thus, measures of covenant protection conceivably could also enter into EAD models, but evidently few have had success quantifying this plausible effect.

Several studies have looked into determinants of EAD but research is generally limited by availability of consistent long term data. Detailed long term studies [22] have found size, collateralisation and maturity to be important determinants. In the author's experience, other than credit-cycle effects discussed below, the amount outstanding and the amount of headroom are the only statistically significant determinants of EAD.

As far as credit-cycle effects, the intuition is not entirely clear. Perhaps, as the LGD data imply, the default threshold is lower in DTs and this gives firms more latitude to increase debt in part by drawing on credit lines. In any case, the empirical evidence for credit-cycle sensitivity is much weaker for EAD than for LGD. Research conducted by authors has shown somewhat positive results, with some but not all samples and for some but not all revolving products indicating a significant credit-cycle effect. Regulatory guidance asks that one consider the possibility and this mechanism of deriving PIT EAD makes it complete and consistent with the rest of PD and LGD counterparts.

However, there is plenty of anecdotal evidence to support the intuition behind the need for PIT EAD. At the time of writing the article, across the industry, oil and gas industry companies were maxing out (See [23], [24]) on their revolving credit facilities (RCFs) which are generally used to cover short term gaps. This does not surprise the authors as **Figure 1** clearly shows that Global Oil and Gas industry is experiencing severe credit conditions which would naturally impact entities drawdown behaviour as they get close to default or try to avoid it altogether. The challenge is to model such behaviour effectively across different type of product types.

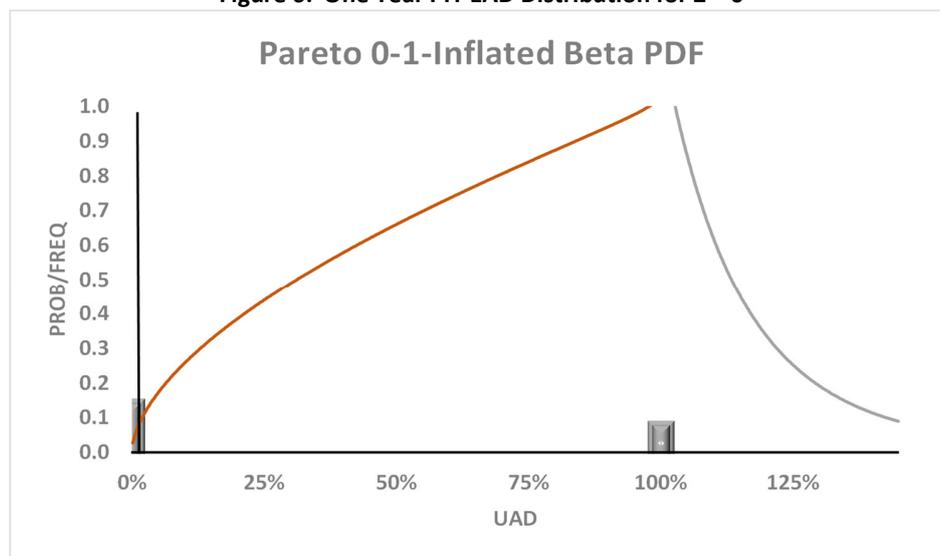
Even more so than with PD and LGD, data development constitutes by far the greatest challenge in EAD modelling. To start, due to the possible influence of an institution's own policies and the absence of large, publicly available samples of EAD data, one may have no alternative to drawing exclusively on the institution's own experience. This limits sample size, which usually forces one to work with broad-based models that combine different facility types into a few, major kinds and place all legal entities under the same models. Further, for some institutions, the matching of outstanding amounts to facilities and limits is problematic, forcing them to apply algorithms that in essence make intelligent guesses. In addition, due to restructuring and replacement of facilities during the run up to default, the matching of facilities prior to and post default may pose challenges. In some cases, one may need to link old facilities to one or more new ones and this may involve manual processes. For contingent facilities, the data may have gaps making it difficult to cleanly identify the sequential development of an exposure from issuance to call to funding. In some of these cases, one may have trouble distinguishing the final cash exposures from other overdraft amounts. Also, multi-obligor and multi-product facilities make it difficult to identify the effective limits and the relevant, draw-down propensities. And finally, limit excess occur particularly as in modelling EAD one utilises at default relative to limits prior to default. In cases of smaller facilities, this can produce explosive values for CCFs and create stability problems in model estimation. One can and must resolve these problems in some fashion. But doing so accounts for the arduousness of EAD modelling.

Turning now to EAD models, as with LGD, analysts have considered a few, different options. Three popular approaches including the simple CCF approach are summarized below (**Table 3**). Our preferred approach applies the zero-one-inflated beta specification, but also appends a Pareto tail in accounting for the occasional case of an EAD in excess of the limit. Just like the LGD counterpart, the movement in Pareto 0-1 Inflated EAD model probability distribution function can be visualized with **Figure 6**. The probability of 1-year EAD = 0% or 100% is indicated by the discrete bars and probability of $0\% < 1\text{-year EAD} < 100\%$ and $1\text{-year EAD} > 100\%$ is given by the probability distribution curve. **Figure 6** refers to the distribution at $Z=0$ and it is easy to visualize how the entire distribution moves left when $Z > 0$ and moves right when $Z < 0$.

Table 3: Selected PIT EAD Model Options

Model Type	Model Description	Model Output	Pros	Cons	Typical Portfolios
CCF	Inputs (e.g. product descriptions, Z factors) determine a CCF from a look-up table; the CCF together with a line's utilisation and limit determine the expected EAD	Point Estimate	Feasible with small samples	$EAD \geq EBD$ restriction typically contrary to facts; no hypothesis tests	All facilities, but material draw-down risk occurs usually only with revolvers
Affine EAD Model	Inputs of product-type, utilisation, Z value, and limit enter into a formula providing a point estimate	Point Estimate	More accurate than CCF model	No hypothesis tests	All facilities, but material draw-down risk occurs usually only with revolvers
Zero-One Inflated Beta with Pareto Tail	Inputs of product type, current utilisation, and Z determine $UAD=0$, $UAD=1$, $UAD > 1$ probabilities, alpha and beta parameters of beta probability density for $0 < UAD < 1$; and delta parameter for Pareto density for $UAD > 1$	PDF	Potentially more accurate than point-estimate models; facilitates hypothesis tests	Calibration requires comparatively large samples; prone to stability problems related to $UAD > 1$ observations	All facilities, but material draw-down risk occurs usually only with revolvers

Figure 6: One Year PIT EAD Distribution for Z = 0



As with LGD, most applications call for EAD expected values. The models providing point estimates produce only these expected values. In the case of the PDF model, one can easily use it to compute the expected values.

5 PIT LGD AND EAD MODELS AND DT LGDs AND EADs

Given PIT LGD and EAD models, one can obtain DT values of LGDs and EADs as needed for RWA by entering into those models together with current values of the other, assumed static inputs a particular, negative Z value for the relevant, industry-region credit-cycle index. If, for example, one interprets DT as meaning “average conditions during official recession periods,” one will find that this implies entering a Z value slightly in below negative one. Indeed, in a stress test in which the relevant Z factor falls from a value of zero to negative 2, one would ordinarily calculate the PIT LGDs and EADs using that negative 2 value. However, for Basel II, setting the Z to an average recession value is the standard convention, since this conforms to the alternative practice of identifying DT parameters with the average values observed in recessions.

Most institutions today have DT LGDs and EADs but no AIRB models sensitive to credit-cycle fluctuations. However, the DT estimates together with one of the PIT models introduced earlier offers a way of inferring PIT parameters. To do this, one accepts the DT values and assumes that they arise from one of the PIT models at a specified DT Z value. Then, for each facility with given DT LGD and EAD, one back solves the chosen models for the consolidated effect of other inputs. One does this so that the models’ expected values at the assumed, DT Z value reconcile with the given, DT values. One then can use those models with the inferred consolidated effect of other inputs to solve for expected LGD and EAD values at other Z values appropriate for PIT estimates. This approach would provide an expeditious way to estimate the PIT parameters needed for stress testing and IFRS 9/CECL prior to developing new, PIT LGD and EAD models.

6 PIT LGD SCENARIOS FOR IFRS 9/CECL AND STRESS TESTING

With models for all PIT PD, LGDs and EADs in hand, one now can now apply them along with models for PIT PDs in creating joint, PD, LGD, and EAD scenarios and thereby PIT loss scenarios. The topic and sub-

topics of deriving PIT loss scenarios deserve detailed sections in themselves but are summarized here briefly. One may accomplish the task of deriving PIT losses by:

- developing Z-factor scenarios in one of the ways discussed below,
- entering the one-period Z changes (ΔZ) from each Z-factor scenario into a multi-threshold, Probit model of transitions from each credit state (grade) to default and to each, non-default state and thereby creating, for each entity with a known, initial, PIT PD scenarios,
- entering the Z values from each Z-factor scenario period-by-period into the PIT LGD and EAD models and thereby obtaining PIT, ELGD (Expected LGD) and EEAD (Expected EAD) scenarios that are jointly determined with the PD ones.

From each joint, PD, ELGD, EEAD scenario, one gets an ECL scenario by forming the period-by-period products: $\Delta PD \times ELGD \times EEAD$. Note that PD denotes a cumulative probability of default and ΔPD represent a one-period, marginal probability of default.

To generate period-by-period PD scenarios from the repeated application of one-period, PIT PD models, one must create scenarios for the one or more inputs into those models. Perhaps the simplest way to accomplish this involves the Probit, credit-factor-conditional, transition-matrix model. This was first introduced to a broad audience by CreditMetrics [25] and it now serves as a mainstay of the regulatory-stress-test processes at many institutions.

One can view the grades or credit states in the model as binned default-distance (DD) measures, with DD representing the single, consolidated input into a Probit PD model. The transition rates to default reflect the PIT PD model. The transition rates to the other states correspond to the stochastic projections of PD-model inputs. Thus, the transition model includes both a PD model and an input-evolution model and so amounts to a tidy way of consolidating all of the needed components. Observe that the transition probabilities vary with ΔZ , which is the one-period change in Z. PIT transitions involve changes in PIT states. Thus, changes in rather than levels of the cycle influence transition rates. In contrast, levels of the cycle influence the ELGDs and EEADs. The process of determining ECL scenarios for stress testing and for IFRS 9/CECL reduces to the generation of Z scenarios. Here one has two broad options:

Statistical scenarios: Surely the simplest way of producing statistical scenarios involves the use of time-series models. These models are commonly used in credit-value-at-risk applications.

One starts with the current and past values of the relevant Zs and obtains scenarios for their stochastic evolution by applying AR1 or AR2 models calibrated to historical experience. In this case, the Z factors evolve under the influence of

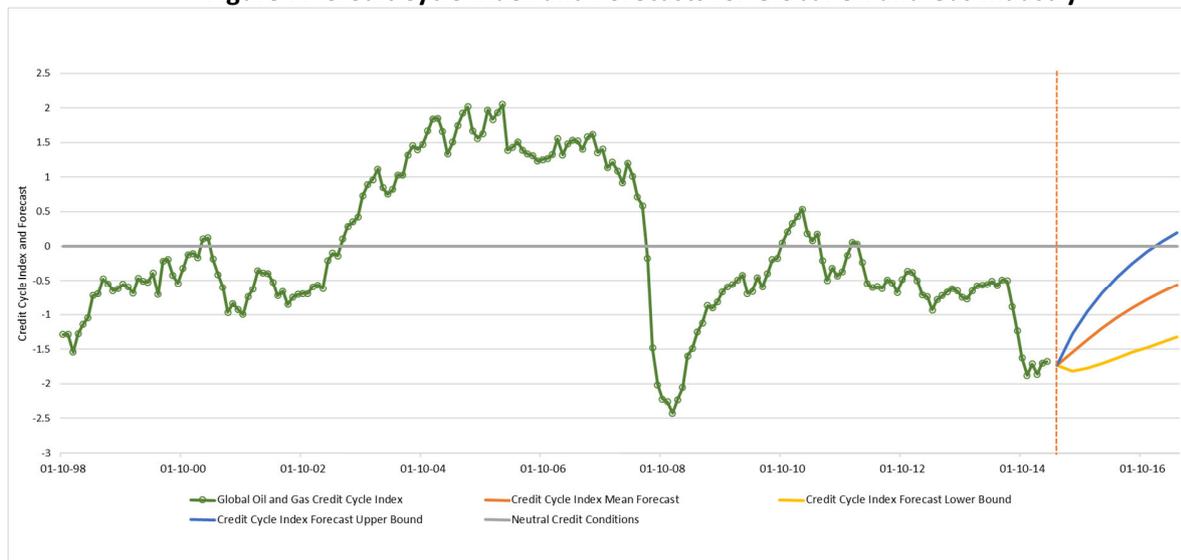
- i) mean reversion, which causes them to gravitate toward zero,
- ii) momentum, which pushes them in the same direction as they moved most recently, and
- iii) most importantly, random shocks.

One can quite easily estimate such models and apply them in generating thousands of scenarios. By averaging the ECLs implied by the joint PD, ELGD, and EEAD scenarios arising from such Z scenarios, one gets a result corresponding to the probability weighted average, ECL term structure that IFRS 9 or CECL demand.

See below Z paths for two sectors with very different initial conditions and so quite different outlooks (**Figure 7** and **Figure 8**). In each case, momentum has a short run effect and mean reversion a persistent

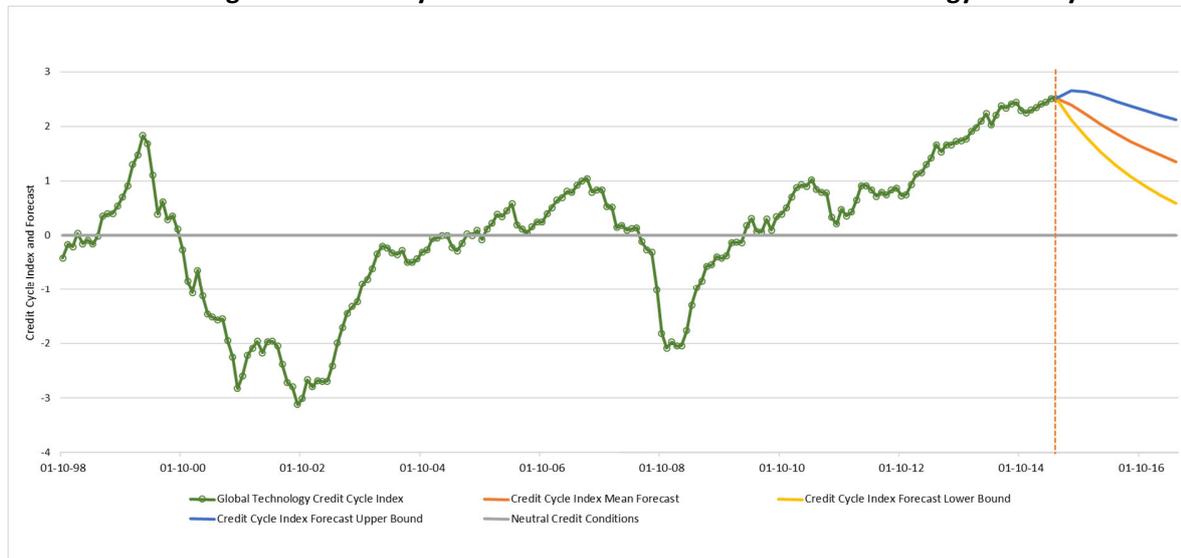
one. Stochastic projections for the next 8 quarters are demonstrated below. Although only three lines corresponding to the period-by-period mean, 95th percentile, and 5th percentile) paths are shown below here, in practice thousands of simulations of $Z_{t, t>today}$ which then drive simulated LGD_t paths are drawn using Monte Carlo Simulation techniques.

Figure 7: Credit Cycle Index and Forecasts for Global Oil and Gas Industry



Sources: Moody's Analytics CreditEdge, AAA models

Figure 8: Credit Cycle Index and Forecasts for Global Technology Industry



Sources: Moody's Analytics CreditEdge, AAA models

Deterministic scenarios: In this case, one assumes that a particular scenario occurs and then derives the credit outcomes implied by it. One may perform this exercise for a handful of distinct scenarios representing, for example, baseline, stress, and severe stress conditions. If these exercises involve pre-defined credit-factor paths, such as Z paths, then the process of determining credit outcomes is straightforward. One merely enters the ΔZ paths into the transition model and the Z paths into the LGD

and EAD models. However, in many cases including most importantly the regulatory stress tests, the predetermined scenarios specify paths not for aggregate-credit factors but for macroeconomic variables. Unfortunately, most of the predetermined macroeconomic variables that the regulators specify have only tenuous relationships to the wholesale credit cycle. Thus, one must develop a further model bridging from the prescribed macroeconomic variables to the credit factors with proximate effects on PDs, LGDs, and EADs. Building a credible bridge model is a non-trivial task and deserves a much lengthy discussion in itself.

One can imagine using the bridge model along with several macroeconomic scenarios as an alternative to the time-series approach to estimating unconditional ECLs. There are few, macroeconomic models available to credit institutions and none, to the author's knowledge, have been used recently in developing large numbers of statistical scenarios. With regard to judgmental scenarios, they have poor track records in part on account of psychological biases.

In closing, the authors observe that, to estimate the probability weighted average of a facility's lifetime credit losses accurately, one must account for the non-linear response of losses to the credit cycle. This non-linear response reflects two phenomena: the convexity of PD functions in the relevant range; and positive PD, LGD, EAD correlation. Only by averaging the ECLs arising from many, joint, PD, LGD, EAD scenarios does one account for these non-linear effects.

7 SUMMARY

Unlike the Basel II rules, which call for the use of a TTC PD along with DT LGDs and EADs in the RWA formula, the regulatory stress test and the new IFRS 9 and proposed CECL accounting standards call for the use of PIT measures. This paper describes approaches for developing PIT LGD and EAD models to be use together with PIT PD ones in developing ECL scenarios either for stress testing or for computing probability weighted average values for PIT ECL term structures. To estimate this probability weighted result accurately, by accounting for non-linear responses of ECL to the cycle, one must run many, joint scenarios for PDs, LGDs, and EADs and average the resulting ECL scenarios.

DECLARATION OF INTEREST

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper.

REFERENCES

1. Basel Committee on Banking Supervision, 2004. International convergence of capital measurement and capital standards: A revised framework. Bank for International Settlements: Basel, June.
2. EBA (2014), "Methodological note EU-wide Stress Test 2014", available at <https://www.eba.europa.eu/documents/10180/669262/Methodological+Note.pdf>
3. IFRS (2015), "Incorporation of forward-looking scenarios" available at <http://www.ifrs.org/Meetings/MeetingDocs/Other%20Meeting/2015/December/ITG/AP1-Forward-looking-scenarios.pdf>
4. Frye, Jon. "Loss given default as a function of the default rate", Chicago Fed publication,
5. Kansas Fed (2014), "Creditor Recovery: The Macroeconomic Dependence of Industry Equilibrium", available at <https://www.kansascityfed.org/publicat/reswkpap/pdf/rwp13-06.pdf>
6. Frye, J. and Jacobs, M., "Credit loss and systematic loss given default", Journal of Credit Risk (1–32) Volume 8/Number 1, Spring 2012

7. Aguais, S. D., Forest, L. R. Jr., Wong, E. Y. L., and Diaz-Ledezma, D. (2004). Point-in time versus through-the-cycle ratings. In *The Basel Handbook: A Guide for Financial Practitioners*, Ong, M. (ed). Risk Books, London.
8. Aguais, S. D., Forest, L. R. Jr., King, M., Lennon, M. C., and Lordkipanidze, B. (2007). Designing and implementing a Basel II compliant PIT–TTC ratings framework. In *The Basel Handbook II: A Guide for Financial Practitioners*, Ong, M.(ed).Risk Books, London.
9. Chawla, G., Forest, L., and Aguais, S. (2013). Deriving point-in-time (PIT) and through the-cycle (TTC) PDs. URL: http://en.wikipedia.org/wiki/Probability_of_default#Throughthe-cycle.28TTC.29_and_Point-in-Time.28PIT.29.
10. Forest L., Chawla G. and Aguais S. D., (2013) “Comment in response to ‘A methodology for point-in-time–through-the-cycle probability of default decomposition in risk classification systems’ by M. Carlehed and A. Petrov”, *The Journal of Risk Model Validation: Volume 7/Number 4, Winter 2013/14*, available at <http://www.risk.net/journal-of-risk-model-validation/technical-paper/2317266/comment-in-response-to-a-methodology-for-point-in-time-through-the-cycle-probability-of-default-decomposition-in-risk-classification-systems-by-m-carlehed-and-a-petrov>
11. Forest L., Chawla G. and Aguais S. D., (2015) “Biased Benchmarks”, *The Journal of Risk Model Validation: Volume 9/Number 2, Summer 2015*, available at <http://www.risk.net/journal-of-risk-model-validation/technical-paper/2412440/biased-benchmarks>
12. Chawla G., Forest L., and Aguais S. D., “AERB: Developing AIRB PIT-TTC PD models using External CRA Ratings”, *The Journal of Risk Model Validation: Volume 9/Number 4, Winter 2015*, available at <http://www.risk.net/journal-of-risk-model-validation/technical-paper/2437473/aerb-developing-airb-pit-ttc-pd-models-using-external-ratings>
13. Acharya, V, Bharath, S T. and Srinivasan, A, (2007), Does industry-wide distress affect defaulted firms? Evidence from creditor recoveries, *Journal of Financial Economics*, 85, issue 3, p. 787-821
14. Chalupka R and Kopecsni J, “Modeling Bank Loan LGD of Corporate and SME Segments: A Case Study”, 2009, Volume 59, Issue 4, pp 360-382.
15. Schuermann, T. (2004) “What do we know about Loss Given Default?”, *Credit Risk Models and Management*, London, UK: Risk Books
16. Carty, L. and D. Lieberman (1996): “Defaulted Bank Loan Recoveries”, Moody’s special comment
17. Hillebran M “Modeling and estimating dependent loss given default”, *Risk*, September 2006, pages 120-125.
18. Hurt, L. a A. Felsovaly (1998) “Measuring loss on Latin American defaulted bank loans, a 27-year study of 27 countries”, *The Journal of Lending and Credit Risk Management*, 80, October, pp. 41-46
19. Thornburn K. (2000) “Bankruptcy auctions: costs, debt recovery and firm survival”, *Journal of Financial Economics*, 58, pp. 337-368
20. Asarnov E. and D. Edwards (1995): “Measuring Loss on Defaulted Bank Loans: A 24 year study”, *Journal of Commercial Lending*, 77(7), pp. 11-23
21. Yashkir O and Yashkir, Y “Loss given default modeling: a comparative analysis”, *The Journal of Risk Model Validation: Volume 7/Number 1, Spring 2013 (25–59)*, available at <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.368.1875&rep=rep1&type=pdf>
22. Jimenez G., Lopes J., and Saurina J, (2008), “Calibrating exposure at default for corporate credit lines”, *Journal of Risk Management in Financial Institutions*, Vol. 2, 2 121–129
23. <http://www.nasdaq.com/article/new-lender-worry-oil-firms-max-out-credit-lines-20160211-01451>
24. <http://www.wsj.com/articles/new-lender-worry-oil-firms-max-out-credit-lines-1455236012>
25. Gupton G., Finger C., and Bhatia M, *Creditmetrics* (1997), “Creditmetrics”, available at https://www.msci.com/resources/technical_documentation/CMTD1.pdf