Scenario Models Without Point-in-Time, Market-Value Drivers Understate Cyclical Variations in Wholesale/Commercial Credit Losses

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Abstract

Many banks use GDP and other, national-income-and-product-account (NIPA), macroeconomic variables (MEVs) as the main, risk drivers in the wholesale/commercial credit, scenario models that they use in estimating losses under baseline and stress conditions. But, if limited to such drivers, those models would understate differences between baseline and stress losses. To obtain accurate estimates of cyclical variations in credit losses, banks must include fully PIT, market-value-related, risk factors as drivers. Otherwise, the results would understate intertemporal variations related to the credit cycle.

We gauge the underestimation of loss variations by comparing, for a hypothetical portfolio representative of US, commercial-and-industrial (C&I) loans, CCAR-2019, credit-loss scenarios driven alternatively by GDP only and by mark-to-market (MtM) asset values, BBB spreads, and GDP. We refer to the first model as GDP-only and the second as point-in-time (PIT).

Under benign credit conditions in 2018 just before the start of the scenarios, the hypothetical portfolio experiences an annualized, charge-off rate of about 17 bps. In the CCAR, severely-adverse (SA) scenario, with GDP as the only risk driver, the projected, charge-off rate in 2020Q2-2021Q1 reaches 88 bps, about 52 bps over the values in the same quarters in the CCAR, baseline scenario. In the SA scenario, with MtM asset-values, credit spreads, and GDP as drivers, the charge-off rate climbs in 2020 to 230 bps, about 197 bps over the baseline. The PIT model produces a maximum, annual charge-off rate under SA conditions of about 3x the TTC rate and 7x baseline. The ratio of 3x TTC matches the highest value of this ratio in the historical data on US bank, C&I, charge-off rates. The GDP-only model under SA conditions produces a maximum ratio of only 1.14x TTC and 2.4x baseline. Thus, the PIT model estimates loss variations quite accurately and the GDP-only model underestimates them by a wide margin.

The GDP-only and PIT models are identical other than in the choice of MEV drivers. The trials therefore isolate the effects of including versus excluding market-value, MEV drivers. Both models use bridge formulas in translating MEV scenarios into a larger number of industry-region, Z-index scenarios. Both models enter the related, industry-region, Z scenarios into the PD, LGD, and EAD models for each facility in the portfolio and thereby produce the credit-loss scenarios. In both cases, the approaches downstream from the MEV scenarios have PIT components. Thus, the results show that, without market-value, MEV drivers, scenario models that otherwise would be PIT are no longer so. Such non-PIT models underestimate the cyclical variability of losses.

Other deficiencies such as hybrid (less than fully PIT) Z inputs into the PD, LGD, and EAD models would produce non-PIT estimates that also understate variations in credit losses.\(^1\) We’ll address this concern in a forthcoming ZRE Working Paper.

\(^1\) To our knowledge, based on conversations with credit-risk modelers and developers globally and on our experience as reviewers of models at several institutions, only Barclays, the Royal Bank of Scotland (RBS), and DBS bank in Singapore have PIT credit models. At Barclays and RBS, we ran the modeling teams that implemented a framework that produced both fully PIT and fully TTC credit measures. This PIT-TTC Ratings Framework was signed-off under each bank’s Basel II Waiver. DBS has developed its PIT models for wholesale/commercial credit by implementing and licensing Z-Risk Engine (ZRE) jointly with AAA. In the Fall of 2017, DBS adopted ZRE as its strategic IFRS 9 solution for wholesale and commercial credit portfolios, see joint DBS-AAA Z-Risk Engine Press Release, October 9\(^{th}\), 2017. In all three of these cases, the PIT credit-cycle indices (CCIs) integral to the PIT PDs, LGDs, and EADs arise either from ZRE or methods equivalent to those implemented by ZRE. Most other institutions have models involving close-to-TTC inputs. The credit projections under such approaches underestimate cyclical variations due not only to the absence...
Regulators and accounting firms have advised that, in running stress tests and in determining provisions under CECL or IFRS 9, banks need to use PIT models. Otherwise, as described above, the results will be prone to large errors. To our knowledge, few institutions today include, in the wholesale/commercial credit models that they use in provisioning and stress testing, the market-value-related drivers that are necessary for those models to be close to PIT. Also, few banks include fully PIT, Z indices as inputs into PD, LGD, and EAD models. Thus, to obtain accurate, PIT estimates of losses, banks urgently need to enhance their wholesale/commercial credit models.  

Outline of Paper

We start below by explaining the difference between PIT and TTC models. We then describe the models used here in producing loss scenarios alternatively with GDP as the only MEV driver and with MtM asset values, credit spreads, and GDP as MEV drivers. We begin with a general overview of the modeling framework. We then offer more detailed descriptions of the major modeling components. Following this, we display the baseline and SA paths for the selected, MEV drivers and the hypothetical portfolio used in the trials. Last, we present the baseline and SA loss estimates from the alternative models.

PIT versus TTC Models

PIT models track cyclical variations in credit risk. Thus, a PIT default model, solved retrospectively drawing on past values of systematic-credit factors, would, for a large portfolio, produce average PDs that approximate closely the time series of the portfolio’s realized, default rates (DRs). PIT loss-given-default (LGD) and exposure-at-default (EAD) models would, in such retrospective trials, track closely the portfolio-wide, realized LGDs and EADs in each time period. In combination, such PIT models would produce estimates of expected-credit-losses (ECLs) that approximate closely the time series of losses of a large portfolio (Figure 1).

TTC models exclude cyclical variations in credit risk. A TTC default model, solved retrospectively, would, for a large portfolio with unchanging composition in terms of long-run-average, default risk, produce a flat, average-PD series that exhibited none of the cyclical rises and falls of the portfolio’s realized DRs. A TTC, LGD model, solved retrospectively on a large portfolio of facilities that were, on balance, unchanging in terms of collateral coverage, seniority, other structural features, and recovery processes, would produce a flat, average-LGD series.

PIT, TTC, and hybrid (intermediate to PIT and TTC) models differ depending on the cyclicity of their inputs. PIT models involve PIT inputs that move up and down fully in step with the credit cycle. TTC models involve TTC inputs inert to cyclical fluctuations. Hybrid models involve hybrid inputs that move with the cycle, but less than enough to represent the totality of cyclicity.

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of market-value, MEV drivers, as examined in this paper, but also to the use of near-TTC models for PDs, LGDs, and EADs.

2 A bank may accomplish this PIT enhancement quickly by implementing the ZRE application as a monthly batch process drawing on one or more of the listed-company, default models offered by several vendors to create a set of industry and region CCIs (credit cycle indices). Under ZRE, a bank may retain the customized industry-region segmentation that it considers most appropriate for its portfolio. Using ZRE, a bank may also retain its existing credit models, modified, for provisioning and stress testing, by implementing the monthly ZRE batch process along with existing credit models. Thus, ZRE sits on top of a bank’s existing models and so requires no time-consuming redevelopment.
The mathematical formulas that transform inputs into outputs in those three cases often are the same. Thus, in a Probit PD model, one applies the standard-normal, cumulative-probability-distribution function (CDF) in transforming inputs into PDs. The ‘PIT-ness’ of the inputs determines the PIT-ness of the PDs. Thus, the PDs would be PIT, TTC, or hybrid, respectively, depending on whether the inputs were 100% PIT, 0% PIT, or somewhere intermediate to 0% and 100%.

![US Bank C&I Annualized Loss Rates](image)

*Figure 1: PIT and TTC (1990-to-date average) Annualized Loss Rates for US Bank C&I Loans. Source: Board of Governors of the Federal Reserve System.*

[https://www.federalreserve.gov/releases/chargeoff/chgallsa.htm](https://www.federalreserve.gov/releases/chargeoff/chgallsa.htm)

If a bank were to change the long-run-average, default-risk composition of its lending portfolio, then, related to this change in customer business strategy, high-quality PIT and TTC default models would show the same change in average PD. Thus, both models would perform equally well in recognizing non-cyclical changes and so every PIT model has a TTC one within it dealing with non-cyclical variations.

**Models Used Here in Generating CCAR Scenarios**

We describe below the GDP-only and PIT models that we use in running CCAR-2019, loss scenarios. Both start with known or assumed paths of one or more, MEVs. The models convert the MEVs into trendless, credit-risk indices, denoted MEV Zs. Those indices enter into a bridge (or expansion) model, which, on the basis of MEV-Z paths, estimates industry and region, Z paths. The industry and region Zs derive from industry and region, median PDs produced by a listed-company, PIT, default model. Next, the models combine industry and region, Z paths for each permissible industry-region pair and obtain industry-region, Z paths. Those industry-region Zs, entered into PD, LGD, and EAD models for facilities within each, industry-region sector, generate the PD, expected LGD (ELGD), and expected EAD (EEAD) paths that in turn determine the ECL time series implied by each, CCAR scenario. See below a schematic representation of the modeling framework (*Figure 2*).

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3 The ZRE team has worked with all of the listed-company, default models available from vendors. In this case study we’re using the CreditEdge model from Moody’s Analytics.
Rather than entering transformed MEVs directly into PD, LGD and EAD models, the approach here includes the intermediate step of bridging to industry and region Zs. This occurs for a couple of reasons. To start, the industry and region Zs offer information on differences among sectors in initial, credit conditions. These initial differences help explain variations among sectors in the future evolution of credit conditions. Further, past research shows that, if added to wholesale/commercial credit PD, LGD, and EAD models, the industry-region Zs dramatically improve model performance in tracking cyclical variations in PDs, LGDs, and EADs. Absent this bridge step, the projected changes in credit conditions in all industries within the US would be the same. Such unduly uniform projections would be inferior to those that account for the predictable differences that trace to variations in initial, industry and region Zs.

In each case, conditional on a MEV scenario, the projections amount to a sequence of one-quarter estimates of the defaults and losses experienced by a portfolio with fixed, TTC attributes. The estimates of defaulted EADs and ECLs enter into perpetual inventories of recovery exposures and identified impairments. Then, under an assumed, exit rate, set to 25% quarterly in these trials, the identified impairments transition into charge-offs.

Assuming an unchanging risk appetite, the static-TTC-attribute assumption offers a tractable way of projecting a bank’s “good book” of credit facilities. Under this regulatory convention, the facilities in the portfolio have maturities, limits, utilization rates, and TTC values of PDs, LGDs, and EADs that remain the same in every quarter during a scenario. Fluctuations over time in quarterly ECLs trace entirely to varying credit-cycle conditions as gauged by industry and region Zs.

Since each loss scenario amounts to a sequence of single-step estimates, the models in this paper draw only on the default column of the conditional, quarterly, transition matrix (TM) in Figure 2 above. In a multi-step problem, such as in estimating provisions under CECL or IFRS 9, one would use the entire TM.

Industry and Region Zs
The industry and region Zs (Aguais et. al. 2007) derive from PDs produced by a PIT, default model covering listed companies. In this paper, we draw on the Moody’s CreditEdge model. Other vendors of such models include Kamakura, S&P, Bloomberg, and the Credit Research Initiative of the National University of Singapore.
We derive the industry and region, Z indices by

- calculating, for each of the selected industries and regions, the time series of median PDs,
- applying, to each median-PD series, the inverse-normal function and then multiplying by negative one, thereby obtaining a series of median, default distance (DD) measures,
- subtracting the 1990-to-date average value of median DDs, thereby deriving a default-distance gap (DDGAP), cyclical indicator, and
- dividing by the standard deviation of either annual (annual scaling) or quarterly (quarterly scaling) changes in the DDGAPs, thereby producing Z indices.

In some cases, this process yields Zs that exhibit a trend, evidently reflecting accreting changes in the TTC composition of the underlying, listed-company sample. In such cases, we estimate a linear time trend and subtract it from the Zs derived as described above. This yields an entirely cyclical, trendless series.

In running quarterly PD scenarios using conventional formulas, which assume that one-period-changes in credit factors have unitary variance, we apply the quarterly-scaled, Z indices. For all other purposes, we use the annual-scaled versions. Such scaling differences are unimportant for LGD and EAD estimation. LGDs and EADs occur at a point in time (the default time) and vary depending on the Z value at that time. Defaults in contrast occur over a time interval and vary depending on the Z change during that interval.

The inverse-normal transformation assumes that the underlying PD model is Probit. If instead the PD model were logit, we’d start with median, log-odds ratios. If the model were exponential, we’d work with natural logarithms of the PDs.

GDP-Only Model’s MEV Transformation and Bridge Formula
This model has one MEV driver – GDP. The model transforms GDP into a credit-cycle, Z index, denoted ZG. This involves

- forming a first-order-autoregressive (AR(1)) moving average of past, quarterly, GDP values,
- taking natural logarithms of the quarterly ratios of GDP to the moving-average of past GDPs,
- subtracting the 1990-to-date, average value of the logarithmic ratios, thereby producing a cycle-gap measure, and
- dividing by the standard deviation of annual changes in the cycle-gap measure.

Past, ZG values show moderate, cyclical declines in 1990-91 and 2001-02 and a large one in 2007-08 (Figure 3).

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4 Most banks have scenario models drawing on multiple MEVs. However, in some cases, the several MEVs represent GDPs in different countries or other MEVs highly correlated with GDP. Since the portfolio in this case involves only obligors operating principally in the US, the US-GDP case represents the simplest instance of scenarios driven either by country-specific GDPs or by MEVs highly correlated with GDP.
Figure 3: US ZG Series for 1990Q1 to 2018Q4. Authors calculations using GDP data from the Bureau of Economic Analysis of the Department of Commerce obtained at https://fred.stlouisfed.org/series/GDP

Dividing by a moving average approximates the effect of forming the ratio of GDP to non-financial-corporate liabilities. In this process, GDP represents a business cash flow or profit indicator. Thus, the ratio to a moving average provides a proxy measure of cash flow or profits to liabilities. Over 1990Q1-2018Q4, an AR(1) moving average of GDP with coefficient of 0.207 achieves the maximum correlation coefficient of about 99.5% with US, non-financial-corporate liabilities.

After deriving ZG projections from a GDP scenario, the model applies a formula for bridging from ZGs to industry and region Zs. See below (Table 1) the formula that results from regressing past, quarterly changes in industry and region Zs on lagged values of industry and region Zs, lagged values of quarterly changes in industry and region Zs, and contemporaneous and lagged values of quarterly changes in ZG. For simplicity, we assume the same model for all industries and regions and use a pooled sample in estimation.

Table 1: Regression Results for GDP-Only Bridge Model

<table>
<thead>
<tr>
<th>Variable Type</th>
<th>Variable Description</th>
<th>Point Estimate</th>
<th>Est Std Error</th>
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<tbody>
<tr>
<td>Dependent</td>
<td>Industry or Region Z Quarterly Change</td>
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<td>NM</td>
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<td>Industry or Region Z Lagged</td>
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<td>ZG Quarterly Change</td>
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<td>Goodness of Fit</td>
<td>R-Squared</td>
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Sources: Authors’ calculations using Z-Risk Engine formulas, CreditEdge data from Moody’s Analytics, and GDP data from the Bureau of Economic Analysis of the US Department of Commerce.
PIT Model’s MEV Transformations and Bridge Formula

This model has three, MEV drivers: stock prices, credit spreads, and GDP. The model transforms each of these MEVs into Z indices. The GDP transformation occurs as described above for the GDP-only model. The stock-price and credit-spread transformations occur as described below.

The model derives an asset-value Z, denoted ZA by

1. forming an AR(1) moving average of quarterly, S&P500, stock-price-index values,
2. taking natural logarithms of one plus the quarterly ratios of the stock-price index to its moving-average,
3. subtracting the 1990-to-date, average value of the logarithmic ratio, thereby producing a cycle-gap measure, and
4. dividing by the standard deviation of annual changes in the cycle-gap measure.

The ZA series shows a small cyclical decline in 1991 and large drops in 2001-02 and 2007-08 (Figure 4).

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**Figure 4: US ZA Series for 1990Q1 to 2018Q4.** Authors’ calculations using S&P 500 price series obtained at [https://finance.yahoo.com/quote/%5EGSPC/history?p=%5EGSPC](https://finance.yahoo.com/quote/%5EGSPC/history?p=%5EGSPC)

Division by a moving average approximates the effect of forming the ratio of MtM equity to non-financial-corporate liabilities. Then, by adding one, the equity/liability ratio becomes an asset-value/liability one. Over 1990Q1-2018Q4, an AR(1) moving average of the S&P index with coefficient of 0.07 achieves the maximum correlation coefficient of about 99.7% with US, non-financial-corporate liabilities.

The model derives the credit-spread Z, denoted ZS, by

1. taking the negative of the inverse normal of the BBB spread divided by 0.6, thereby obtaining a DD indicator,
2. calculating an AR(1) moving average of the DD series, using an AR(1) coefficient of 0.07,
3. deducting the moving averages from the DDs, thereby obtaining a DDGAP series, and
4. subtracting the 1990-to-date average value of the DDGAP series and dividing by the standard deviation of annual changes in the DDGAP series, thereby producing a Z series.
The ZS series shows a small cyclical decline in 1991, modest drops in 1997-98 and 2000-01, and a very large fall in 2007-08 (Figure 5).

![ZS US](image)

**Figure 5: US ZS Series for 1990Q1 to 2018Q4.** Authors’ calculations drawing on Moody’s seasoned Baa bond yields obtained at [https://fred.stlouisfed.org/series/BAA](https://fred.stlouisfed.org/series/BAA) and 10-year Treasury yields obtained at [https://fred.stlouisfed.org/series/DGS10](https://fred.stlouisfed.org/series/DGS10)

After deriving the ZG, ZA, and ZS projections implied by a scenario for GDP, stock prices, and BBB spreads, the model applies a formula for bridging from the three, MEV Zs to industry and region Zs. The bridge formula results from regressing past, quarterly changes in industry and region Zs on lagged values of industry and region Zs, lagged values of quarterly changes in industry and region Zs, and contemporaneous and lagged values of quarterly changes in ZG, ZA, and ZS (Table 2). Again, as with the GDP-only approach, we assume a common model for all sectors and use a pooled sample in estimation.

**Table 2: Regression Results for PIT Bridge Model**

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<th>Variable Type</th>
<th>Variable Description</th>
<th>Point Estimate</th>
<th>Est Std Error</th>
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<td><strong>Dependent</strong></td>
<td>Industry or Region Z Quarterly Change</td>
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<td>NM</td>
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<td>Industry or Region Z Lagged</td>
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<td>ZS Quarterly Change Lagged</td>
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<td>ZG Quarterly Change</td>
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<td>ZG Quarterly Change Lagged</td>
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<tr>
<td><strong>Goodness of Fit</strong></td>
<td>R-Squared</td>
<td>50%</td>
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Sources: Authors’ calculations using Z-Risk Engine formulas, CreditEdge data from Moody’s Analytics, and GDP data from the Bureau of Economic Analysis of the US Department of Commerce.
Remaining Components Common to the Two Approaches
We now describe the common, downstream components of the GDP-only and PIT approaches. These components draw on the industry and region Zs coming out of the bridge models and, in the end, deliver the ECLs implied by the MEV scenarios.

As noted earlier, combined, industry-region Zs enter as inputs into the PD, LGD, and EAD models. The formula (1) below produces the industry-region-composite, Z indices.

\[
Z_{I,R,t} = \frac{w_is_iZ_{I,t} + (1-w_i)s_RZ_{R,t}}{\sqrt{w_i^2s_i^2 + (1-w_i)^2s_R^2 + 2w_i(1-w_i)\rho_{I,R}s_is_R}}
\]  

(1)

Here I denotes an index identifying an industry, R an index identifying a region, \(Z_{I,R,t}\) the Z value at time t for the composite of industry I and region R, \(Z_{I,t}\) the Z value at time t for industry I, \(Z_{R,t}\) the Z value at time t for region R, \(w_i\) the optimal weight, based on historical estimation of listed-company, DD changes, for combining industry-I’s DDGAPs with and regional ones, \(s_i\) the historical standard deviation of one-period changes in industry-I DDGAPs, \(s_R\) the historical standard deviation of one-period changes in region-R DDGAPs, and \(\rho_{I,R}\) the correlation coefficient between industry-I and region-R, one-period Z changes.

The numerator in (1) is a weighted average of industry and region DDGAPs. The denominator is the standard deviation of one-period changes in the industry-region DDGAP. Owing to the limited size of the samples, which make industry-region cross-tabs impractical, we use weighted averages of separate, industry and region indices as estimates of industry-within-region indices.

The trials apply Probit-PD models, Tobit-LGD models, and Probit-CCF models. For each facility, we draw on the relevant, industry-region Z in translating the PDs, LGDs, and credit-conversion factors (CCFs) from TTC to PIT values as of each scenario quarter.

Each quarterly, conditional PD arises from the formula (2)

\[
PD|Z_{I,R,t}^Q = \Phi \left( -\frac{DD_{TTC} + DDGAP_{I,R,t}^Q}{\sqrt{1 - \rho_{I,R}^Q}} \right) = \Phi \left( -\frac{-\Phi^{-1}(PD_{TTC}) + \sqrt{\rho_{I,R}^Q} (Z_{I,R,t}^Q - Z_{n,I,R}^Q)}{\sqrt{1 - \rho_{I,R}^Q}} \right)
\]  

(2)

Here, \(PD|Z_{I,R,t}^Q\) denotes the PD in the quarter ending at time t, conditional on \(Z_{I,R,t}^Q\), which is the industry-region Z, scaled for a quarterly model, at time t. \(\Phi\) represents the standard normal, cumulative distribution function, \(PD_{TTC}\) the obligor’s, quarterly TTC PD, \(\rho_{I,R}^Q\) the industry-region, systematic factor’s proportion of overall, quarterly, \(\Delta DD\) variance, and \(Z_{n,I,R}^Q\) the relevant, quarterly, Z-norm value.

Z-norm accounts for the convexity of the PD function. To calculate the conditional, PIT PD in a quarter, one needs to enter into the Probit function the expected value of DD (PIT) at the end of the quarter. One may obtain this DD as the TTC DD plus the end-of-quarter DDGAP. This DDGAP is the same as the square root of rho times the end-of-quarter Z. Following a familiar convention, one may start from the TTC PD. This PD arises as a long-run average over many cyclical settings. Due to the PD-function’s convexity, this TTC PD exceeds the PD conditional on DD being at its TTC value. Thus, the negative of the inverse normal of the TTC PD will fall short of the TTC DD. How much less? By the square root of rho times Z-norm.
Indeed, this is the way we define Z-norm. Consequently, rearranging components, one finds that the numerator in the far right in (2) produces the needed result -- the TTC DD plus the end-of-quarter, DDGAP.

Conditional ELGDs under the Tobit model arise from the formula (3). Recall that the LGD model using quarterly scaling produces the same result as the model using annual scaling. The calculations in the trials use the annually-scaled model as depicted below.

\[
ELGD|Z^0_{i,R,t} = ELGD|Z_{i,R,t} = \Phi \left( \frac{1-m}{s} \right) + m \left( \Phi \left( \frac{1-m}{s} \right) - \Phi \left( \frac{-m}{s} \right) \right) + s \left( \Phi \left( \frac{-m}{s} \right) - \Phi \left( \frac{1-m}{s} \right) \right)
\]

\[
m = m_0 + mZ_{i,R,t}
\]

\[
s = \exp(s_0 + sZ_{i,R,t})
\]

\[
m_0 = \text{back solved based on TTC ELGD}
\]

\[
m_Z = -0.04
\]

\[
s_0 = -0.91
\]

\[
s_Z = -0.06
\]

Here \(\Phi\) (lower-case \(\Phi\)) denotes the standard-normal, density function, \(m\) the Tobit central tendency, \(s\) the Tobit standard deviation, \(\exp\) the exponential function, and \(m_0, m_Z, s_0,\) and \(s_Z\) parameters in the functions determining \(m\) and \(s\). We’ve set the values of all parameters other than \(m_0\) to values obtained on average in past research. Then the TTC LGD input determines \(m_0\). In detailed, LGD modeling, the \(m_0\) and \(s_0\) parameters would arise as functions of facility structural features such as collateralization and seniority. But such structural effects are assumed static, unaffected by credit-cycle conditions and already subsumed in the TTC LGD. Thus, these features need not appear explicitly in these trials.

The ELGD function is close to linear in the relevant range. Thus, no Z-norm adjustment occurs.

Conditional ECCFs arise from formula (4).

\[
ECCF|Z^0_{i,R,t} = ECCF|Z_{i,R,t} = \Phi \left( c_0 + cZ_{i,R,t} \right)
\]

\[
c_0 = \text{back solved based on TTC ECCF}
\]

\[
c_Z = -0.04
\]

The -0.04 value for \(c_Z\) reflects past EAD modeling results for RCFs. Conditional EADs result from formula (5).

\[
EEAD|Z^0_{i,R,t} = EEAD|Z_{i,R,t} = L \cdot (EU + ECCF|Z_{i,R,t} \cdot (100\% - EU))
\]

Here, \(L\) denotes the facility limit, which is constant due to the static-portfolio assumption.

The conditional, PD, LGD, and EAD estimates of conditional determine the conditional ECL estimates as follows.

\[
ECL|Z^0_{i,R,t} = PD|Z^0_{i,R,t} \cdot ELGD|Z^0_{i,R,t} \cdot EEAD|Z^0_{i,R,t}
\]
The ECL estimates in turn produce estimates of charge-offs and identified impairments (formula (7)).

\[
CO_t = \frac{1}{pr\_dur} \cdot M_{t-1} \\
M_t = M_{t-1} + ECL_t - CO_t
\]

\(M_t\) denotes identified impairments at time \(t\), \(CO_t\) charge-offs at time \(t\), and \(pr\_dur\) the average duration of identified impairments, set to four quarters in these trials.

The estimates of book values of loans come from the following formulas.

\[
V_t = V_{G,t} + V_{B,t} \\
V_{G,t} = V_G = \sum_f EU_f L_f \\
V_{B,t} = \left(1 - \frac{1}{pr\_dur}\right) V_{B,t-1} + \sum_f PD_{f,t} EEAD_{f,t}
\]

Here \(V_t\) denotes the book value of loans, \(V_{G,t}\) the value of the good book, \(V_{B,t}\) the value of the bad (impaired loan) book, and \(f\) a facility identifier. Due to the static-portfolio assumption, the value of the good book remains constant. We compute it as the sum of the products of limits, \(L_f\), and expected utilization rates, \(EU_f\). The model calculates each quarterly charge-off rate as \(CO_t/V_{t-1}\).

**Industry-Region Sectors and Credit Portfolio Used in the Trials**

In the trials, we use 20, industry-regional groupings in varying proportions of the total, lending limit (Table 3). Within each industry-region subset of the portfolio, the facilities have static, TTC attributes (Table 4).

### Table 3: Industry-Region Sectors Used in the Scenarios

<table>
<thead>
<tr>
<th>Industry</th>
<th>Region¹</th>
<th>Proportion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aerospace and Defense</td>
<td>North America (non-Fi)</td>
<td>1%</td>
</tr>
<tr>
<td>Banking</td>
<td>North America Fi</td>
<td>5%</td>
</tr>
<tr>
<td>Basic Industries</td>
<td>North America (non-Fi)</td>
<td>5%</td>
</tr>
<tr>
<td>Business and Consumer Services</td>
<td>North America (non-Fi)</td>
<td>20%</td>
</tr>
<tr>
<td>Chemicals and Plastic Products</td>
<td>North America (non-Fi)</td>
<td>2%</td>
</tr>
<tr>
<td>Construction</td>
<td>North America (non-Fi)</td>
<td>10%</td>
</tr>
<tr>
<td>Consumer Products</td>
<td>North America (non-Fi)</td>
<td>2%</td>
</tr>
<tr>
<td>Finance, Insurance, and Real Estate</td>
<td>North America Fi</td>
<td>10%</td>
</tr>
<tr>
<td>Hotels and Leisure</td>
<td>North America (non-Fi)</td>
<td>5%</td>
</tr>
<tr>
<td>Machinery and Equipment</td>
<td>North America (non-Fi)</td>
<td>3%</td>
</tr>
<tr>
<td>Media</td>
<td>North America (non-Fi)</td>
<td>5%</td>
</tr>
<tr>
<td>Medical</td>
<td>North America (non-Fi)</td>
<td>5%</td>
</tr>
<tr>
<td>Mining</td>
<td>North America (non-Fi)</td>
<td>1%</td>
</tr>
<tr>
<td>Motor Vehicles and Parts</td>
<td>North America (non-Fi)</td>
<td>5%</td>
</tr>
<tr>
<td>Oil and Gas</td>
<td>North America (non-Fi)</td>
<td>3%</td>
</tr>
<tr>
<td>Retail and Wholesale Trade</td>
<td>North America (non-Fi)</td>
<td>6%</td>
</tr>
<tr>
<td>Metals</td>
<td>North America (non-Fi)</td>
<td>4%</td>
</tr>
<tr>
<td>Technology</td>
<td>North America (non-Fi)</td>
<td>4%</td>
</tr>
<tr>
<td>Transportation</td>
<td>North America (non-Fi)</td>
<td>3%</td>
</tr>
</tbody>
</table>
Utilities | North America (non-FI) | 1%
---|---|---
All | | 100%

1 North America ≡ US and Canada

**Table 4: Static Attributes of Facilities Within Each Industry-Region Sub-Portfolio**

<table>
<thead>
<tr>
<th>Weight</th>
<th>Entity TTC Grade</th>
<th>Facility Type</th>
<th>Limit $s mm</th>
<th>Primary Region</th>
<th>Primary Industry</th>
<th>EU</th>
<th>PD TTC</th>
<th>ELGD TTC</th>
<th>ECCF TTC</th>
</tr>
</thead>
<tbody>
<tr>
<td>10%</td>
<td>A</td>
<td>RCF</td>
<td>30</td>
<td></td>
<td></td>
<td>10%</td>
<td>0.01</td>
<td>40</td>
<td>75</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TL</td>
<td>30</td>
<td></td>
<td></td>
<td>100%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25%</td>
<td>BBB</td>
<td>RCF</td>
<td>30</td>
<td>North America or North America FI</td>
<td>One of 20</td>
<td>20%</td>
<td>0.03</td>
<td>40</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TL</td>
<td>30</td>
<td></td>
<td></td>
<td>100%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>45%</td>
<td>BB</td>
<td>RCF</td>
<td>30</td>
<td>North America or North America FI</td>
<td>One of 20</td>
<td>30%</td>
<td>0.14</td>
<td>35</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TL</td>
<td>30</td>
<td></td>
<td></td>
<td>100%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15%</td>
<td>B</td>
<td>RCF</td>
<td>30</td>
<td></td>
<td></td>
<td>30%</td>
<td>0.97</td>
<td>45</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TL</td>
<td>30</td>
<td></td>
<td></td>
<td>100%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>CCC</td>
<td>RCF</td>
<td>30</td>
<td></td>
<td></td>
<td>50%</td>
<td>6.84</td>
<td>25</td>
<td>45</td>
</tr>
<tr>
<td></td>
<td></td>
<td>TL</td>
<td>30</td>
<td></td>
<td></td>
<td>100%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100%</td>
<td>All</td>
<td>All</td>
<td>60</td>
<td>All</td>
<td>All</td>
<td>63%</td>
<td>0.56</td>
<td>27</td>
<td>73</td>
</tr>
</tbody>
</table>

1 RCF ≡ revolving credit facility; TL ≡ term loan.
2 EU ≡ expected utilization of credit line other than in default.
3 PD TTC ≡ quarterly TTC PD ≡ quarterly PD conditional on Z = Z-norm.
4 ELGD TTC ≡ expected value of LGD conditional on Z = 0.
5 ECCF TTC ≡ expected value of CCF conditional on Z = 0.
6 Calculated as a weighted sum of limits, with the weights in the left-hand column distributed evenly to the associated, RCF and TL facilities. Thus, in the case of the A-graded obligor, the 10% weight gets distributed 5% to the RCF and 5% to the TL.

**CCAR-2019 Scenarios**

The CCAR-2019 economic assumptions for the MEVs entering into the baseline and SA scenarios in this paper appear below (Table 5). GDP paths enter into the GDP-only model. GDP, credit spread, and MtM-asset-value paths enter into the PIT model. In that PIT model, we represent credit spreads by the BBB corporate-bond yield minus the 10-year Treasury yield. We estimate MtM, asset values on the basis of the S&P 500 stock-price index. In running CCAR scenarios for the S&P index, we assume that it changes at the same rates as the Dow Jones Total Stock Market Index. The CCAR scenarios specify paths for the latter index.

**Table 5: CCAR-2019 Economic Assumptions for MEVs Used in the Trials**

<table>
<thead>
<tr>
<th>Date</th>
<th>Nominal GDP Growth</th>
<th>10-Year Treasury Yield</th>
<th>BBB Corporate Yield</th>
<th>Dow Jones Total Stock Market Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>2018Q4</td>
<td>4.60</td>
<td>3.00</td>
<td>5.00</td>
<td>25,725</td>
</tr>
<tr>
<td>2019Q1</td>
<td>4.20</td>
<td>2.90</td>
<td>4.60</td>
<td>26,026</td>
</tr>
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</table>

Baseline
The two alternative models presented here offer very different estimates of losses under the CCAR-2019 SA scenario. The GDP-only model’s loss projections fall far short of those from the PIT model. Consider first the industry-region Z projections. In the SA scenario, the GDP-only model foresees much less stressful Z outcomes than the PIT model (Figure 6). Consider the weighted average of all industry-region sectors. The GDP-only model projects that, in the SA scenario, the combined Z would bottom out at about 1.35 annual-standard-deviation units below the baseline outlook. This shortfall represents less than half of the 2.93-unit gap projected by the PIT model.

### Alternative CCAR Results

<table>
<thead>
<tr>
<th></th>
<th>GDP-Only</th>
<th>PIT</th>
<th>Combined</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2019Q1</td>
<td>4.80</td>
<td>3.00</td>
<td>4.80</td>
<td>26,367</td>
</tr>
<tr>
<td>2019Q2</td>
<td>4.40</td>
<td>3.10</td>
<td>4.90</td>
<td>26,687</td>
</tr>
<tr>
<td>2019Q3</td>
<td>4.20</td>
<td>3.20</td>
<td>4.90</td>
<td>26,998</td>
</tr>
<tr>
<td>2020Q1</td>
<td>4.00</td>
<td>3.20</td>
<td>4.90</td>
<td>27,299</td>
</tr>
<tr>
<td>2020Q2</td>
<td>4.00</td>
<td>3.20</td>
<td>4.90</td>
<td>27,603</td>
</tr>
<tr>
<td>2020Q3</td>
<td>3.70</td>
<td>3.20</td>
<td>4.90</td>
<td>27,894</td>
</tr>
<tr>
<td>2020Q4</td>
<td>3.80</td>
<td>3.20</td>
<td>4.90</td>
<td>28,193</td>
</tr>
<tr>
<td>2021Q1</td>
<td>4.30</td>
<td>3.40</td>
<td>5.20</td>
<td>28,529</td>
</tr>
<tr>
<td>2021Q2</td>
<td>4.10</td>
<td>3.50</td>
<td>5.10</td>
<td>28,858</td>
</tr>
<tr>
<td>2021Q3</td>
<td>4.10</td>
<td>3.50</td>
<td>5.20</td>
<td>29,191</td>
</tr>
<tr>
<td>2021Q4</td>
<td>4.10</td>
<td>3.60</td>
<td>5.20</td>
<td>29,527</td>
</tr>
<tr>
<td>2022Q1</td>
<td>4.10</td>
<td>3.60</td>
<td>5.20</td>
<td>29,868</td>
</tr>
<tr>
<td>2019Q1</td>
<td>-3.50</td>
<td>0.80</td>
<td>5.30</td>
<td>17,836</td>
</tr>
<tr>
<td>2019Q2</td>
<td>-7.70</td>
<td>0.90</td>
<td>6.10</td>
<td>14,694</td>
</tr>
<tr>
<td>2019Q3</td>
<td>-5.70</td>
<td>1.00</td>
<td>6.50</td>
<td>13,317</td>
</tr>
<tr>
<td>2019Q4</td>
<td>-3.40</td>
<td>1.10</td>
<td>6.50</td>
<td>12,867</td>
</tr>
<tr>
<td>2020Q1</td>
<td>-2.10</td>
<td>1.20</td>
<td>6.20</td>
<td>13,462</td>
</tr>
<tr>
<td>2020Q2</td>
<td>0.50</td>
<td>1.20</td>
<td>5.80</td>
<td>14,421</td>
</tr>
<tr>
<td>2020Q3</td>
<td>1.60</td>
<td>1.20</td>
<td>5.50</td>
<td>15,479</td>
</tr>
<tr>
<td>2020Q4</td>
<td>4.80</td>
<td>1.20</td>
<td>5.10</td>
<td>16,847</td>
</tr>
<tr>
<td>2021Q1</td>
<td>5.40</td>
<td>1.50</td>
<td>5.00</td>
<td>17,788</td>
</tr>
<tr>
<td>2021Q2</td>
<td>5.90</td>
<td>1.60</td>
<td>4.70</td>
<td>19,352</td>
</tr>
<tr>
<td>2021Q3</td>
<td>6.20</td>
<td>1.60</td>
<td>4.40</td>
<td>21,039</td>
</tr>
<tr>
<td>2021Q4</td>
<td>6.40</td>
<td>1.70</td>
<td>4.00</td>
<td>22,940</td>
</tr>
<tr>
<td>2022Q1</td>
<td>6.30</td>
<td>1.80</td>
<td>3.70</td>
<td>25,137</td>
</tr>
</tbody>
</table>

Due to the nonlinearity of the relationship between losses and Zs, the shortfall in the GDP-only-model’s stress outcomes expand when measured as charge-off rates (Figure 7). In the SA scenario, the GDP-only model projects that the charge-off rate in 2020Q2-2021Q1 would rise to about 52 bps over the baseline. This compares with the PIT model’s projection of about 197 bps above the baseline. Starting at 17 bps in 2018, the charge-off rate in the SA scenario reaches an annualized value of 230 bps in 2020 under the PIT model. This peak annual value stands at 3x the TTC rate of 77 bps and over 10x the comparatively low starting value. This reconciles with the historical, US bank, C&I charge-off rates, which in 2009 reached an all-time annual high of 230 bps or about 3x the long-run average and more than 10x the lowest values.

Recall that the two models have the same, PIT components downstream from the MEV scenarios. Thus, the results above demonstrate that, without market-value, MEV drivers, a scenario model that otherwise would be PIT is no longer PIT.

The comparative results above focus on projections errors, assuming accurate, initial estimates in 2018Q4 of industry and region Zs and PIT PDs, LGDs, and EADs. Accurate estimates of initial conditions in
wholesale credit draw on PIT indicators. Banks that exclude such indicators from projections would likely also exclude them from assessments of current conditions. This would produce further errors, which could exacerbate or partly offset projections errors. In any case, a more accurate approach would include market-value-related, PIT measures both in gauging current conditions and in projecting future losses.

Topics for Further Investigation

The models in this study are parsimonious. In both the GDP-only and PIT case, the bridge formula is the same for every industry and region. A more elaborate approach might involve somewhat different drivers and coefficients for each industry or region. For example, the PIT model could draw on a variety of ZA indices, compiled separately for several industries and the non-financial and financial, regional groupings. And both models could draw on GDP and gross-output measures for each of several industries within the US. However, this detail doesn’t appear in the CCAR scenarios and so one would need to draw on more aggregate measures in bridging to these sectors. Further, these more detailed drivers would serve largely as indicators of conditions specific to each of the industry and region, Z groupings. Thus, they would amount to substitutes for the past, industry and region Z values that now appear in the bridge formulas. On this account, we wouldn’t expect that more detailed MEVs would materially alter the results of this study.

As noted earlier, some banks apply hybrid Zs as the systematic-risk inputs into the PD, LGD, and EAD models used in estimating ECLs under stress or baseline conditions. Such an approach may start by imputing historical Zs from past migrations of non-PIT ratings. After that, the approach may involve a formula, estimated using historical data, for projecting those Zs on the basis of forecasts of selected MEVs. This approach delivers hybrid, ECL estimates. Such estimates will understate cyclical variations in losses. We’ll address this deficiency in a future paper.

The results above reveal inadequacies in wholesale-credit-scenario models relying solely on NIPA drivers. To our knowledge, many banks have such models. This raises questions about the continued reliance on such approaches and the oversight provided by model reviewers and regulators. How can such models persist? Do they involve upwardly biased, PD, LGD, and EAD models, which, given under estimates of increases under stress conditions, lead to credible estimates of stress losses? If so, then those same models would produce upwardly biased estimates of baseline losses for CECL or IFRS 9 provisions. So a bank must use fully PIT models to produce accurate estimates of losses under a wide range of credit-cycle conditions.

Summary

As presented in this case study, a credit-scenario model relying solely on GDP as a MEV driver underestimates the increase in C&I, stress losses relative to a baseline by over 60%. Still, many banks use models driven mainly by GDP and possibly other, NIPA MEVs both in stress testing and provisioning. The results here indicate that those banks urgently need to upgrade their wholesale/commercial credit models, introducing market-value-related, PIT drivers.

In both the GDP-only and PIT models examined here, the approaches downstream from the MEV scenarios, including the direct inputs into the PD, LGD, and EAD models, are PIT. Thus, the results show that, without market-value, MEV drivers, scenario models that otherwise would be PIT are no longer so. Such non-PIT models underestimate the cyclical variability of losses.
Other deficiencies such as hybrid (less than fully PIT), direct inputs into the PD, LGD, and EAD models would produce non-PIT estimates that understate temporal variations in credit losses. We’ll address this concern in a forthcoming ZRE Working Paper.

References