Inaccuracies Caused by Hybrid Credit Models and Remedies as Implemented by ZRE

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Abstract

Few banks have wholesale/commercial credit models that are point-in-time (PIT), tracking closely the sometimes large, cyclical variations in default and loss rates.[1] Instead, most models at banks are "hybrid," producing estimates more sensitive to the cycle than cyclically inert, through-the-cycle (TTC) ones, but less sensitive, typically much less so, than PIT ones. The hybrid cases include most probability-of-default (PD) models arising from scorecards and scenario models featuring Z inputs related to hybrid-ratings, transition matrices (TMs). Also, despite ample evidence that LGDs vary cyclically, many banks apply LGD models without systematic links to the credit cycle. While no models produce perfect estimates, PIT models dominate hybrid ones in accuracy and so are essential in stress testing and IFRS-9 or CECL provisioning (see **Figure 1**). Thus, banks urgently need to convert their hybrid models to PIT.

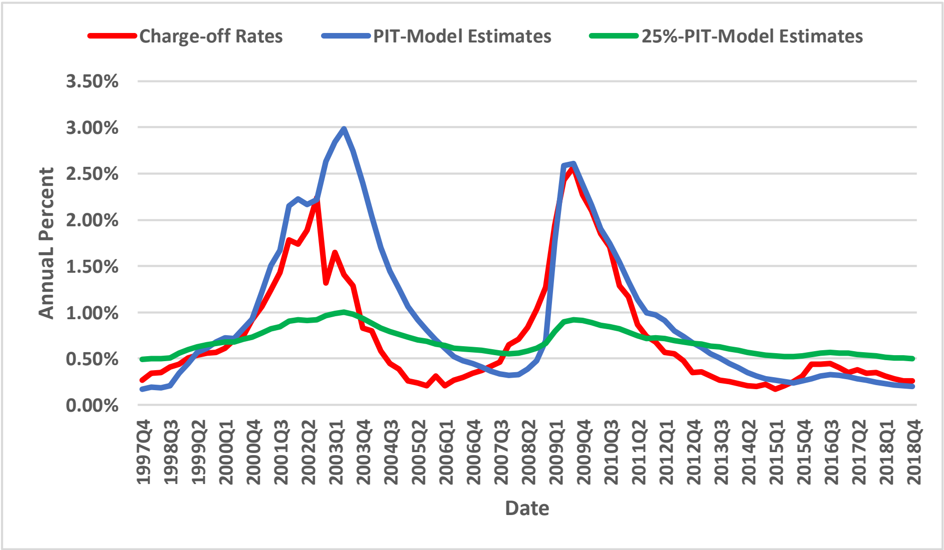

Figure 1: Back Tests Comparing PIT and Hybrid Model Estimates With Actual Values of US-Bank, C&I Charge-Off Rates Over 1997Q4-2018Q4: *Source: Author's calculations using ZRE methods, Moody's CreditEdge data, and US Federal Reserve data found at* https://www.federalreserve.gov/releases/chargeoff/chgallsa.htm

PIT, TTC, and hybrid models differ mainly in the *cyclicality* of their inputs. To convert a hybrid or TTC model to PIT, one must add inputs that, along with existing ones, fully account for the credit cycle. More

[1] To our knowledge, based on conversations with credit-risk modelers and developers globally and on our experience as reviewers of models at several institutions, only Barclays, the Royal Bank of Scotland (RBS), and DBS bank in Singapore have PIT credit models. At Barclays and RBS, we ran the modeling teams that implemented methods for producing both PIT and TTC credit measures. This PIT-TTC Framework was signed-off under each bank's Basel II Waiver. DBS has developed PIT models for wholesale/commercial credit by implementing AAA's, Z-Risk Engine (ZRE) application. In the Fall of 2017, DBS adopted ZRE as its strategic IFRS-9 solution for wholesale and commercial portfolios (see joint DBS-AAA Z-Risk Engine Press Release, October 9th, 2017). In all three of these cases, the PIT credit-cycle indices integral to the PIT PDs, LGDs, and EADs arise either from ZRE or methods equivalent to those implemented by ZRE. Most other institutions have models involving close-to-TTC inputs.

frequent updating of hybrid or TTC inputs would improve estimation of the relative risk of different exposures. But this would have little effect on estimates of cyclical variations in absolute risk.

We estimate below the magnitudes of the inaccuracies arising from credit models with "PIT-ness" of 25%, about the same as S&P and Moody's ratings. PIT-ness measures the cyclical amplitude of a credit model's, consolidated input relative to a PIT input. A PIT model has PIT-ness of 100%; a TTC model 0%; and a hybrid model between 0% and 100%. The hybrid, PD/TM, loss-given-default (LGD), and exposure-at-default (EAD) models in the trials below are all 25% PIT.

For estimates as of 31 Dec 2018, the 25%-PIT, hybrid models overstate the lifetime, expected credit loss (ECL) of a portfolio representative of US, commercial and industrial (C&I) loans by 17%. The over estimation occurs, because credit-cycle conditions were better than average at yearend 2018. Hybrid inputs under represent this beneficial cycle both initially and, with diminishing magnitudes, subsequently over the lifetimes of loans.

In the CCAR-2019, severely-adverse (SA) scenario, with mark-to-market (MtM) asset values, credit spreads, and GDP as macroeconomic-variable (MEV) drivers, the same, hybrid models understate the rise in charge-off rates relative to the baseline by 85%. In this case, the hybrid inputs grossly under estimate the deviations between stress and baseline, credit-cycle conditions.

In quarterly back tests comparing model estimates with actual, charge-off rates on US-bank, C&I loans over 1995-2018, the hybrid models substantially under estimate losses in the 2001-02 and 2008-09 recessions and over state them in most, other periods. Here, the hybrid inputs move up and down cyclically much less than past, PIT inputs. While the models drawing on PIT inputs over state losses during the 2003-06 recovery, they dominate the hybrid models in overall accuracy since 1995.

The results here resemble those in our earlier ZRE paper focusing on errors caused by non-PIT, MEV drivers in scenario models.[2] In the ZRE paper here, the shortfalls in stress relative to baseline losses trace to non-PIT, credit-model inputs. In the earlier paper, the credit-model inputs were PIT, but the shortfalls occurred, because the MEV-based estimates of the inputs had far less cyclicality than the inputs themselves.

To eliminate avoidable inaccuracies, banks could supplement hybrid-model inputs with credit-cycle indices (CCIs) derived from listed-company, PIT PDs. Several vendors offer such PDs. With ample data, one might determine the model sensitivities to the CCIs by re-estimating the models with the CCIs included as additional explanatory variables. With sparse data, one might assign sensitivities the same as those estimated in other, comparable cases. In either event, with the CCIs as inputs, the models would provide substantially more accurate, near-PIT estimates.

Outline of Paper

We start by explaining the way in which hybrid models differ from PIT ones and we present evidence that many banks are struggling with producing PIT estimates. Then we introduce the CCIs that this paper uses in producing PIT outputs. We compare the cyclicality of those CCIs with that of a consolidated, 25%-PIT input. Next, we describe the models that we use in

[2] See https://www.z-riskengine.com/media/1115/zre_scenario_models_without_pit_drivers_understate_wholesale_credit_losses.pdf.

- estimating lifetime ECLs,
- determining baseline and stress losses, and
- conducting back tests.

We also explain the model modifications that lead to 25%-PIT outputs. Following that, we present the trial portfolio, which is structured to be representative of US bank, C&I loans. Next, we describe the trial results, which reveal the inaccuracies caused by hybrid models. Last, we explain how to convert hybrid models to PIT. Appendices include more detailed descriptions of the models in the trials.

Hybrid Inputs Produce Hybrid Outputs

PIT models respond fully to the credit cycle; TTC models respond not at all. Most credit models at banks are hybrid, intermediate to PIT and TTC, but often closer to TTC.

Credit models involve formulas that translate risk inputs into PDs, LGDs, and EADs. The formulas, often cumulative-probability-distribution functions (CDFs), are typically the same regardless of whether the inputs are near PIT or close to TTC. Near-PIT inputs produce near-PIT outputs. Near-TTC inputs produce near-TTC outputs.

Consider a Probit PD model. The standard-normal CDF translates inputs into PDs. If the inputs were PIT, the PDs would be so. If the inputs were hybrid, the PDs would be hybrid.

As another hybrid example, consider the conditional-TM model that several banks use in running credit scenarios. Under an approach introduced by Forest, Belkin, and Suchower (1998), one imputes, for each sector with a sufficiently large ratings sample, a ρ parameter and credit-cycle, Z indices from time series of tabulated TMs. Then, one determines, for each sector with its own TMs, a statistical relationship between MEVs and historical Zs. One uses that relationship in translating MEV scenarios into Z ones. For each sector, those Z scenarios, entered along with ρ into the same TM model used in imputing Zs, produce PD scenarios.

Unfortunately, the ratings underlying TMs are typically closer to TTC than PIT. This includes ratings from S&P and Moody's (Forest L., Chawla G. and Aguais S. D., 2015) and from scorecard models at banks (Levy and Zhang, 2018). Thus, the PDs arising from this TM model will understate cycles in default rates (DRs).

Some mistakenly believe that, to get a PIT model, one merely needs to keep the inputs current. In scenario models, one accomplishes this by estimating the inputs on the basis of the assumed, MEV paths. This doesn't make the results PIT. On the contrary, PIT models differ from hybrid and TTC ones mainly in the cyclicality of inputs, not in the promptness of updating.

Further, one can't repair the underestimation of cyclicality by manually increasing the sensitivities of credit-model outputs to inputs. Those sensitivities arise mainly from an effort to explain the very large, cross-sectional variations in the relative risks of borrowers and exposures and only secondarily from an attempt to explain the smaller, inter-temporal variations. Thus, manually increasing the input sensitivities would impair the overall, cross-sectional and inter-temporal explanation of credit risk. Instead, one needs to supplement existing, mostly relative-risk inputs with cyclical ones that determine the changing magnitudes of broad-based, absolute risk.

Evidence Indicates that Many Banks Today Can't Produce Credible, PIT Estimates

A recent benchmarking study from Global Credit Data (2019) indicates that many banks are unable to produce credible, PIT estimates. The ECL estimates from the participating banks for a common, hypothetical portfolio differ substantially, with third- and first-quartile values varying by 4x. As a further example, the study includes a case in which the banks provide estimates of a five-year, PIT-PD, term structure for a UK borrower in manufacturing with a one-year TTC PD of 75 bps. The reported five-year, PIT PDs vary by a factor of 7!

Industry and Region CCIs Used in the Trials

The models below feature industry and region CCIs as the factors that describe the credit cycle and, when entered as inputs into PD, TM, LGD, and EAD models, yield PIT outputs. These CCIs derive from the listed-company PDs produced by a Merton-type, PIT, default model. In this paper, we draw on the Moody's CreditEdge model. Other vendors of such models include Kamakura, S&P, Bloomberg, and the Credit Research Initiative of the National University of Singapore.

We first develop industry and region, default-distance-gap (DDGAP) indices by

- calculating, for each industry and region, the time series of median PDs,
- applying, to each median PD, the inverse-normal CDF and multiplying the result by negative one, thereby obtaining a series of median, default distance (DD) measures, and
- subtracting the 1990-to-date, average value of median DDs, thereby deriving DDGAPs.

After that, we obtain industry and region, Z indices by

- calculating the standard deviations of annual and quarterly changes in DDGAPs, and
- dividing the DDGAPs by the standard deviations of annual and quarterly changes, thereby producing Zs scaled, respectively, for quarterly and annual models.

In some cases, this process yields DDGAPs and Zs that exhibit a trend, evidently reflecting accreting changes in the TTC composition of the underlying, listed-company sample. If so, we estimate linear time trends and subtract them from the DDGAPs and Zs described above. This yields entirely cyclical, trendless, DDGAP and Z series.

In running quarterly PD scenarios, we enter, into one-quarter-horizon, TM/PD models, quarterly changes in quarterly-scaled, Z indices. An annual PD model would require annually-scaled Zs. Such scaling differences have no effect on LGD and EAD estimation. The related models configured either for annually or quarterly scaled Zs produce the same results. LGDs and EADs occur at a point in time (the default time) and vary depending on the state of the credit cycle at that time. Defaults in contrast occur over an interval of time and, given that the obligor DDs are known as of the start of the interval, vary depending on the *change* in the cycle during that interval. Holding creditworthiness fixed, shorter intervals imply lower PDs and higher values for the levels and one-period changes in DDs, DDGAPs, and Zs.

The PIT and hybrid DDGAPs intrinsic to the trials differ only in cyclical amplitude (**Figure 2**). The changes and levels of hybrid DDGAPs have standard deviations only 25% as great as those of PIT DDGAPs.

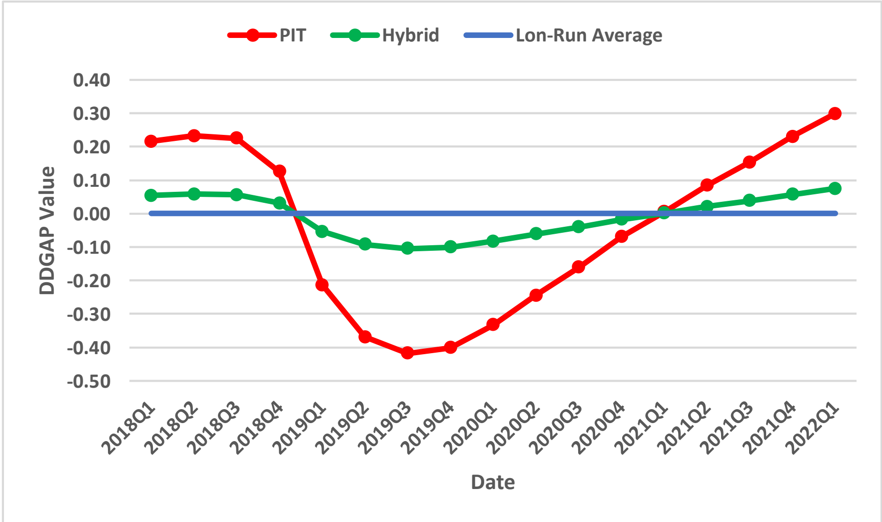

Figure 2: Comparison of PIT and 25%-PIT DDGAPs; *Source: Authors' calculations applying ZRE methods to Moody's Credit Edge data and CCAR-2019 Severely-Adverse-Scenario assumptions*

Models Used in the Trials

The trials below involve three, overlapping models with the same PD/TM, expected LGD (ELGD), and expected EAD (EEAD) functions and the same Z inputs into those functions. The first one (ECL Model) estimates one-year and lifetime ECLs unconditionally, without assuming anything restrictive about the evolution of systematic and idiosyncratic, credit factors. The second (Scenario Model) estimates ECLs conditionally, with systematic, credit factors evolving in the way predicted by assumed paths for selected MEVs. The third (Back Test Model) is the same as the Scenario Model, except that it leaves out the MEV-based estimation of Zs and instead enters, as the systematic-risk inputs into the PD, LGD, and EAD models, actual, historical, Z values.

In each trial, we run the ECL, Scenario, or Back Test Model twice: once formulated to produce PIT estimates and once to produce hybrid, 25%-PIT estimates. We convert from PIT to 25% PIT by multiplying the coefficients applied to the relevant CCIs in the PIT models for TM/PD, LGD, and EAD by 25%. This reduction in coefficient magnitudes has an effect equivalent to reducing the cyclical amplitudes of the credit-cycle inputs. Thus, the ΔZ inputs into the PIT and hybrid, TM/PD models are the same in both the PIT and hybrid runs, but, due to the different coefficients applied to those ΔZs, the outputs are PIT and hybrid, respectively.

Only the CCIs appear explicitly as variables affecting prospective, credit losses. Under conventional assumptions, the inputs explaining borrower- or facility-specific effects either remain constant (LGD and EAD models) or evolve randomly (PD/TM models). Further, the descriptions of the facilities in the trials specify TTC values for PDs, ELGDs, and expected-credit-conversion-factors (ECCFs). These TTC values, along with the assumptions of either constancy or random evolution, pin down the specific effects.

The ECL Model

- runs joint, quarterly, Monte Carlo simulations of the Zs for each valid, industry and region pair,
- combines the Z simulations for each pair and produces industry-region, Z simulations,

- enters those industry-region, Z simulations into conditional, TM, LGD, and EAD models, and thereby derives ECL simulations for each facility in the trial portfolio, and
- averages the quarterly ECL simulations for each facility and, applying each facility's effective interest rate (EIR) in discounting, produces one-year and lifetime ECLs.

The Scenario Model

- starts with assumed, quarterly scenarios for selected MEVs,
- transforms the MEV scenarios into MEV-Z ones,
- enters the MEV-Z scenarios into a bridge model and thereby generates the industry and region, Z scenarios implied by the MEV ones,
- combines the Z scenarios for each, valid, industry and region pair and produces industry-region, Z scenarios,
- enters the industry-region Z scenarios into PD, LGD, and EAD models for each facility and thereby creates quarterly, conditional ECLs for each facility.

The Back Test Model

- draws on past Zs for each of the industry-region sectors, and
- enters the relevant Zs quarterly into the Scenario Model's PD, ELGD, and EEAD functions for each facility in the trial portfolio and thereby estimates, for the portfolio as a whole, past, quarterly values of defaults, specific provisions, charge offs, and charge-off rates.

More detailed descriptions of the ECL and Scenario Model appear in Appendices 1 and 2.

Description of the Trial Portfolio

The trial portfolio includes a mix of sectors (**Table 1**) and facilities (**Table 2**) designed to be representative of US bank, C&I loans. For simplicity, the facilities are the same for each, industry-region sector. The portfolio has an annualized, TTC, charge-off rate of 77 bps. This is the same as the US bank, annualized, C&I, net-charge-off rate on average over 1990-2018.

Table 1: Industry-Region Sectors Used in the Trials

Industry	Region[1]	Proportion
Aerospace and Defense	North America (non-FI)	1%
Banking	North America FI	5%
Basic Industries	North America (non-FI)	5%
Business and Consumer Services	North America (non-FI)	20%
Chemicals and Plastic Products	North America (non-FI)	2%
Construction	North America (non-FI)	10%
Consumer Products	North America (non-FI)	2%
Finance, Insurance, and Real Estate	North America FI	10%
Hotels and Leisure	North America (non-FI)	5%
Machinery and Equipment	North America (non-FI)	3%
Media	North America (non-FI)	5%
Medical	North America (non-FI)	5%
Mining	North America (non-FI)	1%
Motor Vehicles and Parts	North America (non-FI)	5%
Oil and Gas	North America (non-FI)	3%

Retail and Wholesale Trade	North America (non-FI)	6%
Metals	North America (non-FI)	4%
Technology	North America (non-FI)	4%
Transportation	North America (non-FI)	3%
Utilities	North America (non-FI)	1%
All		100%

[1] North America ≡ US and Canada

Table 2: Attributes of Facilities Within Each Industry-Region Sub-Portfolio

Weight	TTC Grade	Facility Type[1]	Limit $s mm	Maturity in Years	EU[2]	PD TTC[3]	ELGD TTC[4]	ECCF TTC[5]	EIR[6]
10%	A	RCF	30	1.0	10%	0.05%	40%	75%	3.00%
		TL	30	1.0	100%		40%	100%	3.00%
25%	BBB	RCF	30	2.0	20%	0.17%	40%	45%	3.30%
		TL	30	2.0	100%		40%	100%	3.30%
45%	BB	RCF	30	3.0	30%	0.72%	35%	45%	4.00%
		TL	30	3.0	100%		35%	100%	4.00%
15%	B	RCF	30	3.0	30%	4.04%	30%	45%	5.00%
		TL	30	3.0	100%		30%	100%	5.00%
5%	CCC	RCF	30	2.0	50%	22.63%	25%	45%	6.00%
		TL	30	2.0	100%		25%	100%	6.00%
100%	All	All	60[7]	2.5	63%	2.11%	27%	73%	3.98%

1 RCF ≡ revolving credit facility; TL ≡ term loan.
2 EU ≡ expected utilization of credit line other than in default.
3 PD TTC ≡ annual TTC PD ≡ annual PD conditional on Z = Z-norm.
4 ELGD TTC ≡ expected value of LGD conditional on Z = 0.
5 ECCF TTC ≡ expected value of CCF conditional on Z = 0.
6 EIR ≡ effective interest rate.
7 Calculated as a weighted sum of limits, with the weights in the left-hand column distributed evenly to the associated, RCF and TL facilities. Thus, in the case of the A-graded obligor, the 10% weight gets distributed 5% to the RCF and 5% to the TL.

In the ECL Model, the attributes in **Table 2** describe the trial facilities as of the start date, 31 Dec 2018. In the Scenario and Back Test models, those attributes describe the performing facilities at the start of each quarter in the scenario or back test.

The ECL Model estimates ECLs over the life of each loan. This multi-step exercise involves the entire TM. The Scenario and Back Test models, in contrast, run a series of one-quarter estimates for a static-TTC-attribute portfolio. This sequence of single-step problems involves only the PD column of the TM.

Results of the Trials

We present below the comparative results of using, respectively, PIT and hybrid models in

- estimating the ECLs needed for establishing provisions under IFRS 9 or CECL,

- running baseline and stress, credit-loss scenarios, and
- conducting back tests over 1997Q4 to 2018Q4.

One-Year and Lifetime ECLs: Hybrid and PIT

Starting on 31 December 2018, the hybrid, 25%-PIT estimates of the trial portfolio's one-year and lifetime ECLs exceed the PIT values by 21% and 17%, respectively (**Table 3**). Credit conditions at the end of 2018 were moderately positive, with a weighted average Z value (annual scaling) of about 0.67. The hybrid model understates the beneficial effect of this above-average setting. While mean reversion in Zs causes the hybrid model's underestimation of cycle effects to diminish over time, the effects during the simulations are still on balance biased down. This downward bias in beneficial, cyclical influences produces an upward bias in PDs and ECLs.

Table 3: One-Year and Lifetime ECLs for the Trial Portfolio as Estimated by PIT and Hybrid Models

As of Date	Model		ECL (% of Book Value)		Ratio Hybrid to PIT	
	Type	PIT-ness	One Year	Lifetime	One Year	Lifetime
31 Dec, 2018	PIT	100%	0.49%	1.26%	1.21	1.17
	Hybrid	25%	0.59%	1.47%		

Source: ZRE application drawing on Moody's CreditEdge data.

The bias caused by hybrid inputs varies with the state of the cycle at the time of lifetime-ECL estimation (**Figure 3**). If the initial Zs and DDGAPs are close to the cycle-neutral value of zero, the bias is small. If initial Z and DDGAP values are substantially positive (negative), the bias is substantially negative (positive). The ECL Model's, Z equations include, as explanatory variables, Z values lagged both one and two quarters. In the estimates summarized in **Figure 3**, the initial Zs are set to the same values in all sectors in both of the two, most recent quarters.

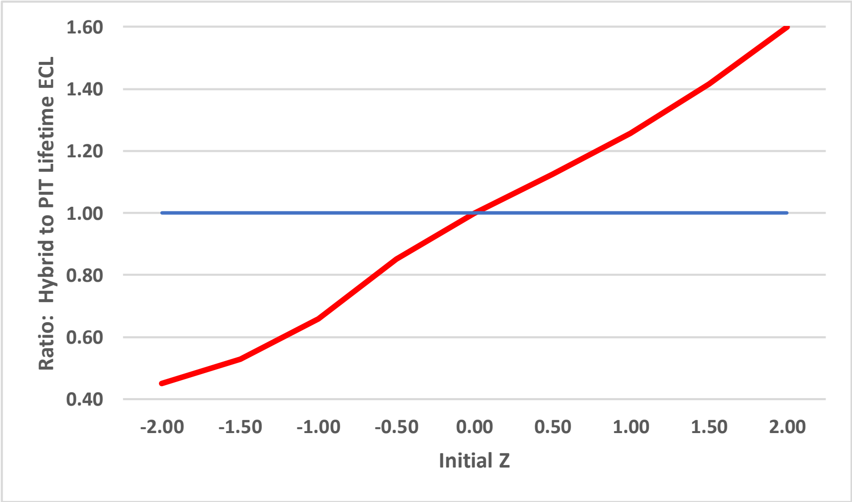

Figure 3: Ratio of Hybrid to PIT, Lifetime ECLs Under Varying, Initial, Credit-Cycle Conditions: *Source: ZRE application drawing on Moody's CreditEdge data.*

As an extreme example, consider the errors in the hybrid estimates as of 2009Q1 when the weighted average, Z-index value was -3.41. For the trial portfolio at that time, the hybrid-model estimates a lifetime ECL of 2.32% of book value. This falls short of the 6.97% PIT estimate by 67%.

The cases just discussed involve over- or under-estimation broadly across all sectors. Other circumstances involve sectoral errors of dissimilar magnitudes and possibly different, arithmetic signs, with this potentially leading to poor, origination and portfolio-management decisions. One sees such non-homogeneity most often in comparing commodity-producing industries with other sectors less sensitive to volatile, commodity prices (**Figure 4**). For example, in 2016Q1, following a sharp drop in crude oil prices to less than $40 per barrel, the Z index for the North American, oil-and-gap industry had a value of *negative* 1.10. At the same time, the Z for North American, technology firms had a value of *positive* 0.55. From **Figure 3** one can see that the 25%-PIT model's estimates of lifetime ECLs in 2016Q1 would have been biased *down* by roughly 35% for oil-and-gas firms and *up* by almost 10% for technology businesses. The almost 45% error in comparative provisioning would have contributed to an unduly favorable assessment of oil-and-gas companies relative to technology firms.

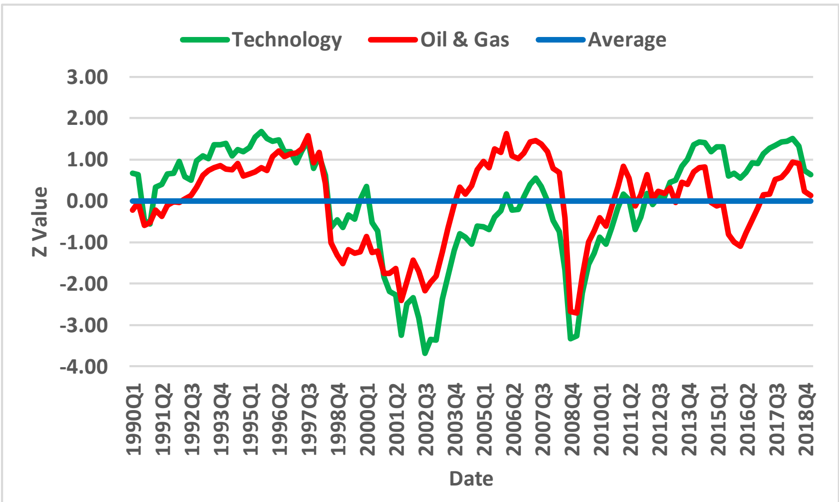

Figure 4: Comparison of North American Z Indices in Technology and Oil & Gas; Source: Authors' calculations using ZRE methods and Moody's, CreditEdge data

CCAR 2019 Severely Adverse Relative to Baseline Results: Hybrid and PIT

The hybrid model understates stress relative to baseline losses by a large amount (**Figure 5**). Under the PIT model with MtM asset values, credit spreads, and GDP as MEV drivers, charge-off rates in the CCAR-2019, severely adverse (SA) scenario rise on balance above baseline values during the 13 scenario quarters by 116 bps. Under the hybrid model with the same, MEV drivers, charge-off rates in the SA scenario rise on balance over baseline values over the entire scenario by 17 bps, about 85% less than PIT values. Under a hybrid model with GDP as the only MEV driver, the average gap between SA and baseline charge-off rates during the scenario interval reaches only 5 bps, an underestimate of almost 95%.

Over the one-year period with the highest, SA, charge-off rates during the scenario, the PIT gap between SA and baseline charge-off rates stands at 199 bps. Under the hybrid model with the same MEV drivers, that gap reaches 28 bps, again about 85% less than the PIT value. Under the hybrid model with only GDP as a MEV driver, the gap is about 11 bps, 95% less than the PIT value.

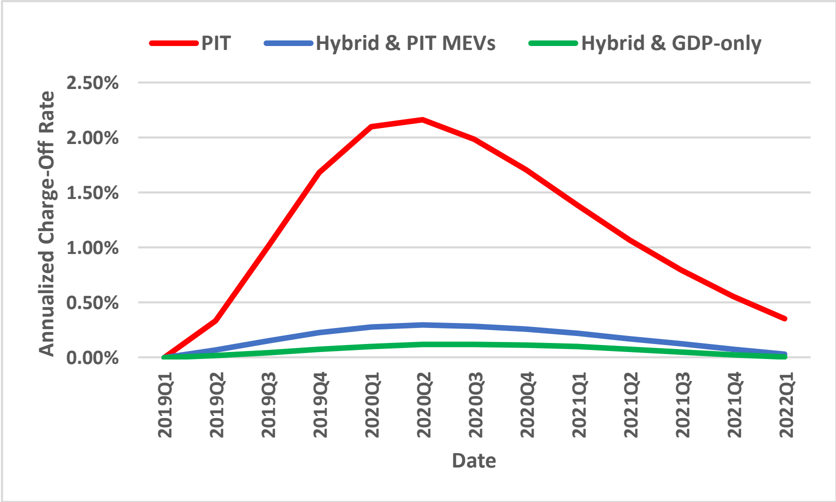

Figure 5: Annualized Charge-Off Rates Relative to Baseline Values: CCAR SA Scenarios Under Alternate Models: *Source: Authors' calculations using ZRE methods and Moody's CreditEdge data.*

Stress scenarios involve cyclical downturns of large size, with Z values falling far into the negative range. Consequently, the hybrid CCIs in stress scenarios include large errors and so the hybrid estimates of PDs and ECLs are highly inaccurate. In contrast, the end-of-2018 estimates of unconditional ECLs involve Z values that start and, in the projections, mostly remain in the moderate range. Thus, the hybrid estimates of PDs and ECLs are moderately inaccurate. However, if today's credit conditions were severely depressed (or extremely strong), the hybrid estimates of unconditional ECLs would by highly inaccurate.

Back Tests Over 1997Q4-2018Q4: Hybrid and PIT

In back tests over 1997Q4-2018Q4, the PIT model explains US, C&I charge-off rates much better than the 25%-PIT, hybrid model (**Figure 1**). These trials involve a sequence of one-quarter projections, each conditional on end-of-quarter values of the industry-region Zs. In the hybrid model, we again mimic inadequate cyclicality of inputs by adjusting down the model coefficients applied to the Z indices. In the PIT case, the trials test the accuracy of the Z indices as cyclical indicators explaining systematic variations in C&I loan charge-offs.

The hybrid model exaggerates losses in good times (2011-2018) and grossly underestimates them in recessions (2001-02, 2008-09). Due to its under representation of the cycle along with the convexity of the relationship between inputs and losses, the hybrid model understates, by 12 bps, the long-run average, annualized, charge-off rate of 77 bps. While the PIT model based on Zs derived from CreditEdge EDFs is imperfect in explaining past, C&I, loss rates, it performs extremely well in tracking the 2008-09 downturn and reconciles almost exactly with the long-run-average charge-off rate. Due in part perhaps to technology companies having greater weight in the CreditEdge sample than in US-bank, C&I loans, the PIT model overstates charge-offs during and after the 2001-02 downturn. But since 2005, the PIT model explains variations in charge-off rates well, much better than the hybrid model.

Converting Hybrid Models to PIT

To convert hybrid or TTC models to near-PIT, one needs to add inputs that track the credit cycle. One may accomplish this by including in the credit models CCIs of the type used in the trials above. We've done this in past work and have found that this sharply improves the goodness-of-fit with the gains occurring in the explanation of inter temporal variations in defaults and losses.

In some cases, banks have too little data to support statistical estimation of a model's PIT-ness, but this need not stop the bank from specifying a credible, PIT model. In the absence of sufficient data, the bank could set the CCI sensitivity of a credit model to that already estimated for another "comparable" model. Suppose, for example, the PDs for a bank's corporate borrowers were inferred from grades established by a process comparable to that used by S&P or Moody's in determining ratings. In this case, one might assume that those PDs had a PIT-ness of 25%, about the same as S&P and Moody's ratings. In that case, the CCIs would have a weight of 75% in the associated, Probit PD model. The ZRE application draws on such PIT-ness parameters, provided in a configuration file, and runs the same regardless of the way one determines the PIT-ness of legacy models.

Summary

Few banks have wholesale/commercial, credit models that include the PIT inputs needed to obtain accurate estimates of defaults and losses. Instead, most models at banks involve hybrid inputs, which move with the cycle, but less so, usually much less so, than PIT ones. Such models produce inaccurate estimates of ECLs.

For a portfolio representative of US bank, C&I loans, we estimate that hybrid, TM/PD, LGD, and EAD models with PIT-ness of 25%, would, as of 31 Dec 2018, overestimate the portfolio's lifetime ECL by 17%. Alternatively, if credit conditions were as in Mar 2009, at the bottom of a deep recession, the hybrid models would underestimate the trial portfolio's lifetime ECL by 67%. Thus, hybrid-model errors are small if credit-cycle factors are close to their long-run average and large if those factors are far above or below average.

In stress scenarios, the hybrid-model errors are very large, because those scenarios assume that credit-cycle conditions decline to far below average. For the portfolio representative of C&I loans, we find that, under the CCAR-2019, SA scenario, the hybrid model understates the rise in charge-off rates over baseline values by about 85%.

In back tests comparing model estimates with realized, C&I loss rates over 1997Q4-2018Q4, the PIT model dominates the hybrid model. The hybrid model grossly under estimates charge-offs in the 2001-02 and 2008-09 recessions and over estimates them in the mostly benign, 2011-2018 period. While not perfect by any means, the PIT model provides much more accurate estimates, particularly in downturns.

Thus, banks urgently need to revamp any models relying on hybrid inputs. This includes most PD models based on scorecard grades and scenario models featuring Z inputs imputed from hybrid-ratings TMs.

Appendix 1: ECL Model

This model runs quarterly Monte Carlo simulations for each valid, industry and region pair. This starts with the random selection of correlated shocks using the formulas (1)

$$\begin{aligned} \varepsilon_{I,t+1} &= u_{1,t+1} \\ \varepsilon_{R,t+1} &= \rho^{\varepsilon}_{I,R} u_{1,t+1} + \sqrt{1-\left(\rho^{\varepsilon}_{I,R}\right)^2} u_{2,t+1} \end{aligned} \tag{1}$$

Here, $\varepsilon_{I,t+1}$denotes the industry innovation at time t+1, $\varepsilon_{R,t+1}$the region innovation at time t+1, $u_{1,t+1}$ a standard-normal, random variable, $u_{2,t+1}$ another standard-normal random variable, and $\rho^{\varepsilon}_{I,R}$ the correlation coefficient between industry and region innovations. The formulas above merely amount to a principal-components or Cholesky decomposition of two, unit-normal, correlated random variables.

The randomly selected innovations and the industry and region Zs in the most recent, past two quarters enter into further simulated values of Zs as determined by the formulas (2)

$$\begin{aligned} Z_{I,t+1} &= z_{I,t+1} + Z_{I,t} \\ z_{I,t+1} &= -m_{1,I} Z_{I,t} + m_{2,I} z_{I,t} + \sigma_I \varepsilon_{I,t+1} \\ Z_{R,t+1} &= z_{R,t+1} + Z_{R,t} \\ z_{R,t+1} &= -m_{1,R} Z_{R,t} + m_{2,R} z_{R,t} + \sigma_R \varepsilon_{R,t+1} \end{aligned} \tag{2}$$

Here $Z_{I,t+1}$ denotes the Z for industry I at time t+1, $z_{I,t+1}$ the one-quarter change in that Z at time t+1, $Z_{R,t}$ the Z for region R at time t, $z_{R,t+1}$ the one-quarter change in that Z at time t+1, $m_{1,I}$ the mean-reversion coefficient for industry I, $m_{2,I}$ the momentum coefficient for industry I, $m_{1,R}$ the mean-reversion coefficient for region R, $m_{2,R}$the momentum coefficient for region R, σ_I the standard deviation of innovations in industry I, and $\sigma_R$the standard deviation of innovations in region R. All of the Zs and parameters in these formulas and the others below involve scaling for a quarterly model. The parameters arise from statistical estimation using historical data.

The model next combines the industry and region Zs for each valid, industry-region pair using the formula (3)

$$Z_{I,R,t} = \frac{w_I s_I Z_{I,t} + (1-w_I) s_R Z_{R,t}}{\sqrt{w_I^2 s_I^2 + (1-w_I)^2 s_R^2 + 2 w_I (1-w_I) \rho_{I,R} s_I s_R}} \tag{3}$$

In (3), $Z_{I,R,t}$ denotes the Z value at time t for the composite of industry I and region R, w_I the optimal weight, based on historical estimation of listed-company, DD changes, for combining industry-I's DDGAPs with regional ones, s_I the historical standard deviation of one-period changes in industry-I DDGAPs, s_R the historical standard deviation of one-period changes in region-R DDGAPs, and $\rho_{I,R}$ the correlation coefficient between industry-I and region-R, one-quarter Z changes.

The model simulates quarterly TMs by entering the quarterly, industry-region Z changes into a Probit TM model (formula (4)).

$$TM(g,h|z_t) = \begin{cases} 1 & if\ g = D, h = D \\ 0 & if\ g = D, h < D \\ \Phi\left(\dfrac{B_{g,D} - \sqrt{\rho} z_t}{\sqrt{1-\rho}}\right) & if\ g < D, h = D \\ 1 - \Phi\left(\dfrac{B_{g,h+1} - \sqrt{\rho} z_t}{\sqrt{1-\rho}}\right) & if\ g < D, h = 1 \\ \Phi\left(\dfrac{B_{g,h} - \sqrt{\rho} z_t}{\sqrt{1-\rho}}\right) - \Phi\left(\dfrac{B_{g,h+1} - \sqrt{\rho} z_t}{\sqrt{1-\rho}}\right) & otherwise \end{cases} \tag{4}$$

Here $TM(g,h|z_t)$ denotes the probability of migrating from integer grade g to h conditional on the indicated, industry-region z value, Φ the standard-normal CDF, D the integer grade corresponding to default, $B_{g,D}$ the (negatively signed) default threshold for grade g, ρ the systematic (z) factor's proportion of the variance of systematic plus idiosyncratic factors, and $B_{g,h}$ the threshold for migration from grade g to h or worse (with higher integer values indicating higher default risk). The PIT values of ρ for each industry-region sector derive from the variances of the related ΔDDGAPs. In the hybrid estimations, we set the value of ρ for each industry-region segment to 0.0625 (= 0.25^2) times the PIT value.

Under the assumption that z has a standard normal distribution, then by integrating out the z dependency in (4) and assuming that historical average migration rates approximate the theoretical averages, one obtains the following formula for the threshold parameters

$$B_{g,h} = \Phi^{-1}\left(\sum_{k \geq h} TM(g,k)\right) \tag{5}$$

In (5) $TM(g,k)$ denotes the historical average migration rate from grade g to k. Thus, one calibrates the thresholds by applying the inverse-normal CDF to the related, partial row sums of long-run average migration rates.

We obtain the long-run-average TM underlying this model by a three-step process. In step one, we form a weighted average of CreditEdge, Moody's, and S&P, long-run average, annual TMs. In step two, we adjust the elements of this average, empirical TM so as to obtain the best fit, subject to constraints, to the average of S&P and Moody's, long-run average, default rate term structures by grade. The constraints in the optimization ensure that the PDs rise as ratings get worse, that all TM cells have positive values, and that total, up and down migration rates by grade remain close to empirical values. Last, we derive a generator for the step-two TM and use that in forming a quarterly, long-run-average TM.

By multiplying the quarterly, conditional TMs in a simulation and selecting from the default column of the resulting product matrices, the model calculates PDs over varying terms for each initial grade g.

$$\begin{aligned} PD(g, t+n | z_{t+1}, \ldots, z_{t+n}) &= \left[\prod_{m=1}^{n} TM(z_{t+m})\right][g, D] \\ TM(z_{t+m}) &= \left[\quad TM(g,h|z_{t+m}) \quad\right] \end{aligned} \tag{6}$$

In (6), $PD(g, t + n|z_{t+1}, \dots, z_{t+n}))$ denotes the cumulative probability of default from time t to t + n under the indicated z scenario for an obligor with an initial grade of g, Π cumulative, matrix multiplication, and $TM(z_{t+m})$ the TM conditional on the given value of the systematic factor at time t+m. Observe that the bracketed term [g, D] to the right of the bigger brackets in the top line of (6) denotes selection of the value in the g^{th} row and D^{th} column of the matrix formed as a product of single-period TMs. This cell contains the cumulative probability of default for an initial grade of g.

At this point, the model has simulated, PD term structures for each of 16, discrete grades. We extend that resolution to the continuum of all possible values of initial creditworthiness by using spot, one-year PDs in interpolating between 18, discrete, PD term structures. The 18 settings include the 16 grades appearing in the TM plus a default (D = 100% PD) grade and a risk-free (no default = 0% PD) grade.

In almost all cases, an obligor has a spot PD that falls between the spot PDs of two of the 18, discrete grades. If so, to derive the obligor estimate, one interpolates between the pair of term structures in a simulation for the grades with spot PDs just above and below the obligor's, spot PD. As an example, suppose that the obligor's, spot PD at a one-year horizon sits half way between the one-year, spot PDs for model grades 5 and 6. Then, accepting the spot PD as the relevant indicator for this interpolation, we'd estimate the obligor's cumulative-PD term structure by averaging the grade-5 and grade-6 term structures.

More generally, we form a weighted average of the term structures of the two grades, A and B, with one-year, spot PDs closest above and below that of the obligor. We determine the weights as follows in (7):

$$\begin{aligned} w_A &= \frac{PD_O^S - PD_B^S}{PD_A^S - PD_B^S} \\ w_B &= 1 - w_A \end{aligned} \tag{7}$$

In (7) w_A denotes the weight on the grade-A term structure, w_B the weight on the grade-B term structure, PD_O^S the one-year spot PD of the obligor, PD_A^S the one-year spot PD of the grade A with spot PD closest above the obligor, and PD_B^S the one-year spot PD of the grade B with spot PD closest below the obligor.

Coming out of the interpolation, the model has simulated, PD term structures for each facility in the trial portfolio. The model then produces the accompanying, jointly simulated, ELGD and EEAD values using the formulas (10), (11), and (12) in Appendix 2 below.

Now, having jointly simulated quarterly values of PDs, ELGDs, and EEADs, the model calculates ECLs as in (8) below:

$$\begin{aligned} ECL(f, t+n) &= \sum_{k=1}^{K} \frac{ECL(k, f, t+n)}{K} \\ ECL(k, f, t+n) &= \Delta PD\left(f, t+n|z_{k,t+1}(f), \dots, z_{k,t+n}(f)\right) \\ &\quad \cdot ELGD\big(f, Z_{k,t+n}(f)\big) \cdot EEAD\big(f, Z_{k,t+n}(f)\big) \end{aligned} \tag{8}$$

Here k indexes simulations and f facilities, which in turn identify obligors, the associated, initial, spot PDs, and the relevant, industry-region sectors. Each facility's obligor's spot PD and sector identify a set of PD and ΔPD simulations. And the facility has other attributes that together with the sector Zs determine the

ELGD and EEAD simulations occurring jointly with the PD ones. By averaging the joint, ΔPD, ELGD, and EEAD scenarios in (8), the model produces the quarterly *unconditional* ECLs for each facility.

Appendix 2: Scenario Model

In each scenario, this model assumes that selected MEVs remain on specified paths and that industry and region Zs remain on the expected-value paths implied by the MEVs. In each scenario, we run the model twice, using PIT and hybrid inputs into the PD model.

The scenario model has (up to) three, MEV drivers: stock prices, credit spreads, and GDP. The model transforms each of these MEVs into Z indices. The stock-price, credit-spread, and GDP transformations occur as described below.

The model derives an asset-value Z, denoted ZA by

- forming an AR(1) moving average of quarterly, S&P500, stock-price-index values,
- taking natural logarithms of one plus the quarterly ratios of the stock-price index to its moving-average,
- subtracting the 1990-to-date, average value of the logarithmic ratio, thereby producing a cycle-gap measure, and
- dividing by the standard deviation of annual changes in the cycle-gap measure.

The ZA series shows a small cyclical decline in 1991 and large drops in 2001-02 and 2007-08 (**Figure 6**).

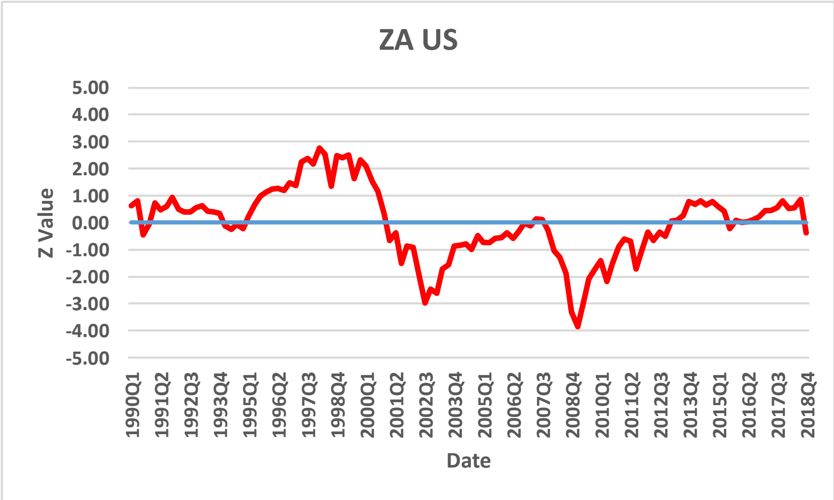

Figure 6: US ZA Series for 1990Q1 to 2018Q4. *Source: Authors' calculations using S&P 500 price series obtained at* https://finance.yahoo.com/quote/%5EGSPC/history?p=%5EGSPC

Division by a moving average approximates the effect of forming the ratio of MtM equity to non-financial-corporate liabilities. Then, by adding one, the equity/liability ratio becomes an asset-value/liability one. Over 1990Q1-2018Q4, an AR(1) moving average of the S&P index with coefficient of 0.07 achieves the maximum correlation coefficient of about 99.7% with US, non-financial-corporate liabilities.

The model derives the credit-spread Z, denoted ZS, by

- taking the negative of the inverse normal of the BBB spread divided by 0.6, thereby obtaining a DD indicator,
- calculating an AR(1) moving average of the DD series, using an AR(1) coefficient of 0.07,
- deducting the moving averages from the DDs, thereby obtaining a DDGAP series, and
- subtracting the 1990-to-date average value of the DDGAP series and dividing by the standard deviation of annual changes in the DDGAP series, thereby producing a Z series.

The ZS series shows a small cyclical decline in 1991, modest drops in 1997-98 and 2000-01, and a very large fall in 2007-08 (**Figure 7**).

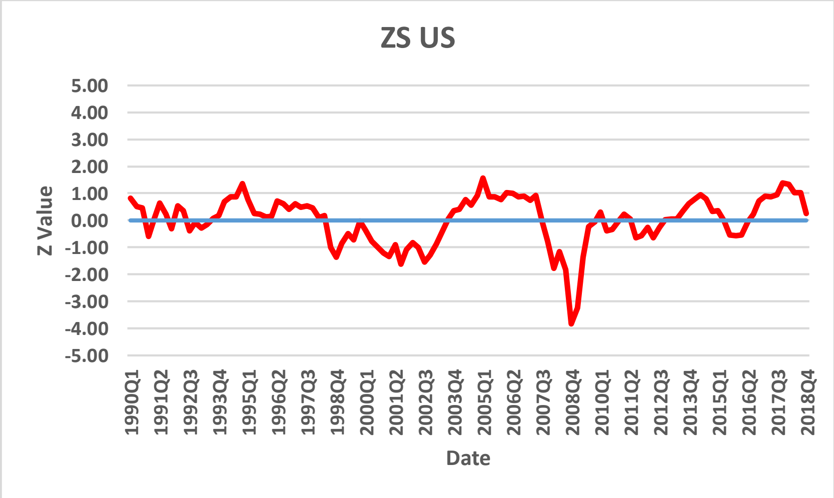

Figure 7: US ZS Series for1990Q1 to 2018Q4. *Authors' calculations drawing on Moody's seasoned Baa bond yields obtained at https://fred.stlouisfed.org/series/BAA and 10-year Treasury yields obtained at https://fred.stlouisfed.org/series/DGS10*

This model derives the GDP Z, denoted ZG, by

- forming a first-order-autoregressive (AR(1)) moving average of past, quarterly, GDP values,
- taking natural logarithms of the quarterly ratios of GDP to the moving-average of past GDPs,
- subtracting the 1990-to-date, average value of the logarithmic ratios, thereby producing a cycle-gap measure, and
- dividing by the standard deviation of annual changes in the cycle-gap measure.

Past, ZG values show moderate, cyclical declines in 1990-91 and 2001-02 and a large one in 2007-08 (**Figure 8**).

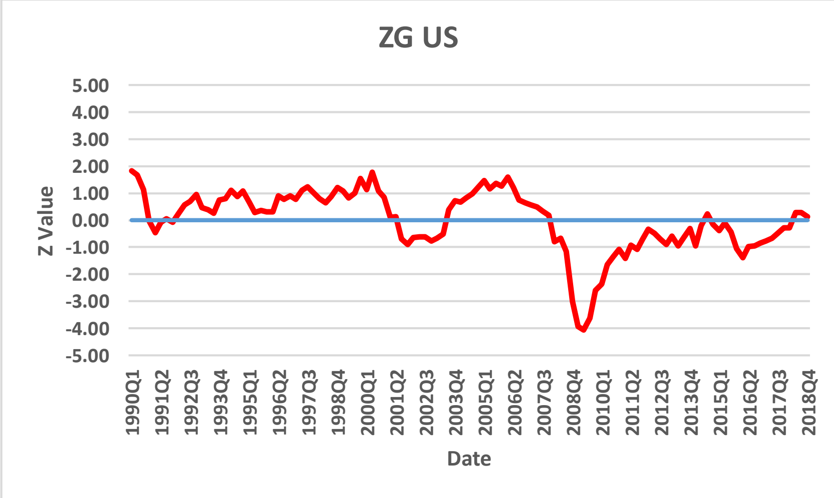

Figure 8: US ZG Series for 1990Q1 to 2018Q4. *Authors calculations using GDP data from the Bureau of Economic Analysis of the Department of Commerce obtained at* https://fred.stlouisfed.org/series/GDP

Dividing by a moving average approximates the effect of forming the ratio of GDP to non-financial-corporate liabilities. In this process, GDP represents a business cash flow or profit indicator. Thus, the ratio to a moving average provides a proxy measure of cash flow or profits to liabilities. Over 1990Q1-2018Q4, an AR(1) moving average of GDP with coefficient of 0.207 achieves the maximum correlation coefficient of about 99.5% with US, non-financial-corporate liabilities.

After deriving the ZA, ZS, and ZG projections implied by a scenario for stock prices, BBB spreads, and GDP, the model applies a formula for bridging from the three, MEV Zs to industry and region Zs. The bridge formula results from regressing past, quarterly changes in industry and region Zs on lagged values of industry and region Zs, lagged values of quarterly changes in industry and region Zs, and contemporaneous and lagged values of quarterly changes in ZG, ZA, and ZS (**Table 4**). We assume a common model for all sectors and use a pooled sample in estimation.

Table 4: Regression Results for PIT Bridge Model

Variable Type	Variable Description	Point Estimate	Est Std Error
Dependent	Industry or Region Z Quarterly Change	NM	NM
Explanatory	Industry or Region Z Lagged	-0.05	0.00
	Industry or Region Z Quarterly Change Lagged	0.09	0.02
	ZA Quarterly Change	0.39	0.01
	ZA Quarterly Change Lagged	0.07	0.02
	ZS Quarterly Change	0.22	0.02
	ZS Quarterly Change Lagged	0.00	0.02
	ZG Quarterly Change	0.03	0.02
	ZG Quarterly Change Lagged	0.01	0.02
Goodness of Fit	R-Squared	50%	

Sources: Authors' calculations using Z-Risk Engine formulas, CreditEdge data from Moody's Analytics, and GDP data from the Bureau of Economic Analysis of the US Department of Commerce.

We also run scenarios with only GDP as a MEV driver. In this case, we apply the bridge formula (**Table 5**) below also obtained by pooled estimation.

Table 5: Regression Results for GDP-Only Bridge Model

Variable Type	Variable Description	Point Estimate	Est Std Error
Dependent	Industry or Region Z Quarterly Change	NM	NM
Explanatory	Industry or Region Z Lagged	-0.07	0.01
	Industry or Region Z Quarterly Change Lagged	0.15	0.02
	ZG Quarterly Change	0.31	0.02
	ZG Quarterly Change Lagged	-0.04	0.02
Goodness of Fit	R-Squared	17%	

Sources: Authors' calculations using Z-Risk Engine formulas, CreditEdge data from Moody's Analytics, and GDP data from the Bureau of Economic Analysis of the US Department of Commerce.

The bridge model produces industry and region Zs. However, combined, industry-region Zs enter as inputs into the PD, LGD, and EAD models. These industry-region Zs arise by means of formula (3) presented above.

The trials apply Probit-PD models, Tobit-LGD models, and Probit-CCF models. For each facility, we draw on the relevant, industry-region Z in translating the PDs, LGDs, and credit-conversion factors (CCFs) from TTC to PIT values as of each scenario quarter.

Each quarterly, conditional PD arises from the formula (9)

$$PD|Z_{I,R,t} = \Phi\left(-\frac{DD_{TTC} + DDGAP_{I,R,t}}{\sqrt{1-\rho_{I,R}}}\right) = \Phi\left(-\frac{-\Phi^{-1}(PD_{TTC}) + \sqrt{\rho_{I,R}}\left(Z_{I,R,t} - Z_{n,I,R}\right)}{\sqrt{1-\rho_{I,R}}}\right) \quad (9)$$

Here, $PD|Z_{I,R,t}$denotes the PD in the quarter ending at time t, conditional on $Z_{I,R,t}$, which is the industry-region Z, scaled for a quarterly model, at time t. Φ represents the standard normal, CDF, PD_{TTC} the obligor's, quarterly TTC PD, $\rho_{I,R}$ the industry-region, systematic factor's proportion of overall, quarterly, ΔDD variance, and $Z_{n,I,R}$ the relevant, quarterly, Z-norm value. In the hybrid estimation, the ρ parameter has a value of 0.0625 (= $0{,}25^2$) times the PIT value.

Z-norm accounts for the convexity of the PD function. To calculate the conditional, PIT PD in a quarter, one needs to enter into the Probit function the expected value of DD (PIT) at the end of the quarter. One may obtain this DD as the TTC DD plus the end-of-quarter DDGAP. This DDGAP is the same as the square root of rho times the end-of-quarter Z. Following a familiar convention, one may start from the TTC PD. This PD arises as a long-run average over many cyclical settings. Due to the PD-function's convexity, this TTC PD exceeds the PD conditional on DD being at its TTC value. Thus, the negative of the inverse normal of the TTC PD will fall short of the TTC DD. How much less? By the square root of rho times Z-norm. Indeed, this is the way we define Z-norm. Consequently, rearranging components, one finds that the numerator in the far right in (9) produces the needed result -- the TTC DD plus the end-of-quarter, DDGAP.

Conditional ELGDs under the Tobit model arise from the formula (10). Recall that the LGD model using quarterly scaling produces the same result as the model using annual scaling. The calculations in the trials use the annually-scaled model as depicted below.

$$\begin{aligned}
ELGD|Z_{I,R,t} \quad &= \Phi\left(-\frac{1-m}{s}\right) + m\left(\Phi\left(\frac{1-m}{s}\right) - \Phi\left(-\frac{m}{s}\right)\right) \\
&+ s\left(\phi\left(-\frac{m}{s}\right) - \phi\left(-\frac{1-m}{s}\right)\right) \\
m \quad &= m_0 + m_Z Z_{I,R,t} \\
s \quad &= exp\left(s_0 + s_Z Z_{I,R,t}\right) \\
m_0 \quad &= backsolved\ based\ on\ TTC\ ELGD \\
m_Z \quad &= -0.04 \\
s_0 \quad &= -0.91 \\
s_Z \quad &= -0.06
\end{aligned} \tag{10}$$

Here ϕ (lower-case Φ) denotes the standard-normal, density function, m the Tobit central tendency, s the Tobit standard deviation, *exp* the exponential function, and m_0, m_Z, s_0, and s_Z parameters in the functions determining *m* and *s*. We've set the values of all parameters other than m_0 to values obtained on average in past research. Then the TTC LGD input determines m_0. In detailed, LGD modeling, the m_0 and s_0 parameters would arise as functions of facility structural features such as collateralization and seniority. But such structural effects are assumed static, unaffected by credit-cycle conditions and already subsumed in the TTC LGD. Thus, these features need not appear explicitly in these trials. The coefficient values above reflect results of past estimations of Tobit LGD models. In the hybrid runs, we set the values of m_Z and s_Z to 25% of the values shown above.

The ELGD function is close to linear in the relevant range. Thus, no Z-norm adjustment occurs.

Conditional ECCFs arise from formula (11).

$$\begin{aligned}
ECCF|Z_{I,R,t} \quad &= \Phi\left(c_0 + c_Z Z_{I,R,t}\right) \\
c_0 \quad &= backsolved\ based\ on\ TTC\ ECCF \\
c_Z \quad &= -0.04
\end{aligned} \tag{11}$$

The -0.04 value for c_Z reflects past EAD model results for RCFs. In the hybrid estimations, we set that coefficient value to -0.01 (= 0.25 x (-0.04)). Conditional EADs result from formula (12).

$$EEAD|Z_{I,R,t} \quad = \mathrm{L} \cdot \left(\mathrm{EU} + ECCF|Z_{I,R,t} \cdot (100\% - EU)\right) \tag{12}$$

Here, L denotes the facility limit, which is constant due to the static-portfolio assumption.

The conditional, PD, LGD, and EAD estimates of conditional determine the conditional ECL estimates as follows.

$$ECL|Z_{I,R,t} \quad = PD|Z_{I,R,t} \cdot ELGD|Z_{I,R,t} \cdot EEAD|Z_{I,R,t} \tag{13}$$

The ECL estimates in turn produce estimates of charge-offs and identified impairments (formula (14)).

$$\begin{aligned} CO_t &= \frac{1}{pr\_dur} \cdot SP_{t-1} \\ SP_t &= SP_{t-1} + ECL_t - CO_t \end{aligned} \tag{14}$$

CO_t denotes charge-offs at time t, SP_t specific provisions at time t, and pr_dur the average duration of identified impairments, set to four quarters in these trials.

The estimates of book values of loans come from the following formulas.

$$\begin{aligned} V_t &= V_{G,t} + V_{B,t} \\ V_{G,t} &= V_G = \sum_f EU_f \, L_f \\ V_{B,t} &= \left(1 - \frac{1}{pr\_dur}\right) V_{B,t-1} + \sum_f PD_{f,t} \, EEAD_{f,t} \end{aligned} \tag{15}$$

Here V_t denotes the book value of loans, $V_{G,t}$ the value of the good book, $V_{B,t}$the value of the bad (impaired loan) book, and f a facility identifier. Due to the static-portfolio assumption, the value of the good book remains constant. We compute it as the sum of the products of limits, L_f, and expected utilization rates, EU_f. The model calculates each quarterly charge-off rate as CO_t/V_{t-1}.

This paper runs the CCAR-2019 baseline and SA scenarios. The baseline and SA MEV assumptions underlying these trials appear below (**Table 6**).

Table 6: CCAR-2019 Economic Assumptions for MEVs Used in the Trials

Date	Nominal GDP Growth	10-Year Treasury Yield	BBB Corporate Yield	Dow Jones Total Stock Market Index
2018Q4	4.60	3.00	5.00	25,725
	Baseline			
2019Q1	4.20	2.90	4.60	26,026
2019Q2	4.80	3.00	4.80	26,367
2019Q3	4.40	3.10	4.90	26,687
2019Q4	4.20	3.20	4.90	26,998
2020Q1	4.00	3.20	4.90	27,299
2020Q2	4.00	3.20	4.90	27,603
2020Q3	3.70	3.20	4.90	27,894
2020Q4	3.80	3.20	4.90	28,193
2021Q1	4.30	3.40	5.20	28,529
2021Q2	4.10	3.50	5.10	28,858
2021Q3	4.10	3.50	5.20	29,191
2021Q4	4.10	3.50	5.20	29,527
2022Q1	4.10	3.60	5.20	29,868
	Severely Adverse			
2019Q1	-3.50	0.80	5.30	17,836
2019Q2	-7.70	0.90	6.10	14,694
2019Q3	-5.70	1.00	6.50	13,317
2019Q4	-3.40	1.10	6.50	12,862
2020Q1	-2.10	1.20	6.20	13,462
2020Q2	0.50	1.20	5.80	14,421
2020Q3	1.60	1.20	5.50	15,479

2020Q4	4.80	1.20	5.10	16,847
2021Q1	5.40	1.50	5.00	17,788
2021Q2	5.90	1.60	4.70	19,352
2021Q3	6.20	1.60	4.40	21,039
2021Q4	6.40	1.70	4.00	22,940
2022Q1	6.30	1.80	3.70	25,137

Source: Board of Governors of the Federal Reserve System. Data transcribed from https://www.federalreserve.gov/newsevents/pressreleases/files/bcreg20190213a1.pdf

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