

Convexity and Correlation Effects in Expected Credit Loss calculations for IFRS9/CECL and Stress Testing

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ABSTRACT

In this study, the authors demonstrate that the convexity of PD functions together with correlation among PD, LGD, and EAD outcomes impart skewness to the credit-loss, probability-distribution function (PDF) and thereby increase the expected values of credit losses (ECLs) by as much as 20% or more according to estimates presented later. With regard to convexity, the magnitude of the effect on ECLs depends on the amount of convexity in PD functions as well as the extent of the random dispersion in the credit-risk factors that affect PDs. With regard to correlation, the magnitude of the effect depends on the amount of correlated variation in PD, LGD, and EAD outcomes. In accounting for these effects, one may apply credit-cycle indices in modifying the existing PD, LGD, and EAD models so that they produce point-in-time (PIT) estimates that move together over time in the way implied by common, credit-cycle effects. Having done

that, an institution can account for convexity and correlation effects in producing the unbiased estimates of ECLs needed in determining losses under stress scenarios or impairments under IFRS 9/CECL.

Keywords: Point-in-Time (PIT), Through-the-cycle (TTC), IFRS9/CECL, Expected Credit Loss (ECL), Stress Testing, Correlation, non-linear losses

1. OVERVIEW

The Basel II Advanced Internal Ratings Based (AIRB) approach ^[1] inspired financial institutions to develop models for PD, LGD and EAD and to use them for RWA calculations. In contrast, IFRS9/CECL and Stress Testing require calculations of Expected Credit Losses (ECLs) using several scenarios. Financial institutions must assure that the (i) data, (ii) PD, LGD, EAD model specification and (iii) ECL formulation lead to the unbiased ECLs as required by regulatory and accounting requirements.

Paragraph 65 of EBA's stress-testing-methodology document ^[2] states that, in all credit-risk-related calculations except RWA for all portfolios and not just AIRB ones, institutions should use PIT measures that reflect the current outlook for business-cycle conditions. IFRS 9 requirements, paragraph 5.5.17, ^[3] state that ECL estimates should:

- correspond to unbiased, probability-weighted averages as determined by evaluating a range of possible outcomes, and
- draw on reasonable and supportable information that is available without undue cost or effort at the reporting date about past events, current conditions, and forecasts of future economic conditions.

The starting point for ECL should be PIT PD, LGD, EAD. In previous research ^{[4], [5], [6], [7], [8], [9], [9]} credit cycle indices have been used to convert hybrid/TTC like PD, LGD and EAD models prevalent in wholesale/corporate/commercial credit modelling space into PIT ones. As a result, these unconditional PIT PD/LGD/EAD are an unbiased, unconditional estimate of default rate/loss/exposure over any specified horizon. PIT ECL is derived in an analogous fashion. The authors recommend that unconditional expectation of the future drawing on today's unconditional PIT measures should be used for IFRS/CECL and conditional expectation on a specific future state should be used for Stress Testing.

In this study, the authors demonstrate the presence of two effects:

- (i) convexity of PIT PD function and
- (ii) correlation between PIT PD, LGD and EAD.

Convexity of PIT PD probability distributions arises due to non-linear functional form between PIT PD and credit cycle index in a manner that negative credit shocks lead to higher PIT PD compared to what a linear model would predict. Use of a single or small number of forward looking scenarios can lead to underestimation of ECL and thus not meeting IFRS 9 "unbiased probability weighted" requirement. Submissions in IFRS 9 Transition Resource Group document ^[3] have expressed similar concerns due to such non-linear losses.

In this study, the authors provide an insight into convexity effects, which imply that to fully incorporate the related effects in ECL, institutions need to make use of several (more than one or a few) scenarios covering the entire span of credit conditions at all times over the life of a facility. Use of the entire domain of credit conditions along with convex functions, leads to an entire range of PIT PD including those in the extreme tail end, leading to unbiased nature of ECLs. Such credit index scenarios also need to be weighted according to their probability of occurrence.

Correlation between PIT PD, LGD and EAD has been documented in research and the authors provide further evidence of it. Such correlation effects are best understood as arising from the common influence of systematic credit risk factors. The authors show that one may use industry-region credit cycle indices obtained from large samples of listed-company PIT PDs in deriving PIT PDs, LGDs, and EADs across broad based wholesale portfolios of exposures related to unlisted as well as listed companies.

Using illustrative data, this article finds that accounting for convexity and correlation effects raises ECLs by about 20%-30%, an important amount in light of the regulatory and accounting requirements of unbiased measure of ECLs. As a remedy, one may draw on credit cycle index scenarios in producing the required joint PD, LGD, and EAD scenarios that lead to unbiased estimates of ECLs.

2. CONVEXITY EFFECTS IN PIT PD

By convexity we refer to the non-linear functional form of PD model with respect to the systematic risk factor. By convexity, we mean that in the general range of ‘good book’, i.e. a TTC PD of few bps to few percentage points, a negative systematic credit shock leads to a much bigger change in PDs compared to a positive credit shock of same magnitude. One finds that these effects are particularly severe in the range of values common in PD models and not so much in LGD and EAD models. This is illustrated using a case of a single time period during which default may occur, a single, credit-risk factor (Z) distributed normally, with an expected value for the coming period of zero, and a Probit PD model, which, for any given Z value, determines the associated, Z-conditional PD.

The standard Vasicek formulation ^[11] for PD model which is a good starting point for PD modelling is

$$PD_{i,t} = \Phi \left(- \frac{\Phi^{-1}(p_{i,t}) + \sqrt{\rho} \cdot Z_t}{\sqrt{1-\rho}} \right) \quad (1)$$

where $PD_{i,t}$ is the Probability of Default for the i^{th} entity at time t
 Φ - standard normal cumulative distribution function
 Z_t is systematic factor (credit cycle index)
 ρ – correlation factor related systematic factor Z

Since this is well known to most practitioners, then why do practitioners sometimes forget the convexity of functions and mistakenly assuming $E[PD] = \Pr(PD(Z=0))$? From the formulation above, it is clear that the unconditional expectation of PD (with respect to all, possible Z values) should be greater than the PD conditional on Z being at its expected value (of 0). Graphically, this is illustrated using **Error! Reference source not found.**, where a normally distributed credit cycle index (Z) creates a convex function of PD.

Figure 1: PD as convex function of Z (Illustrative data)

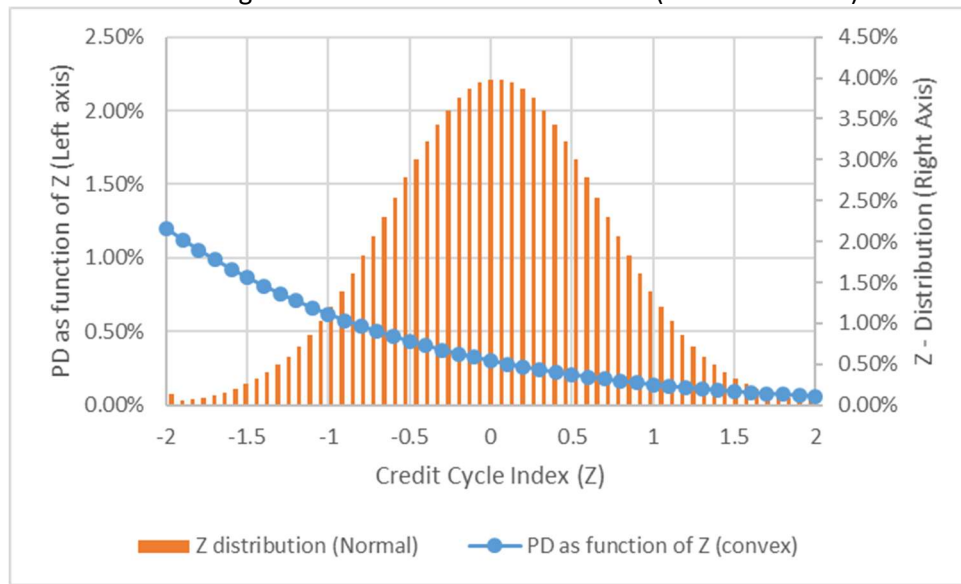
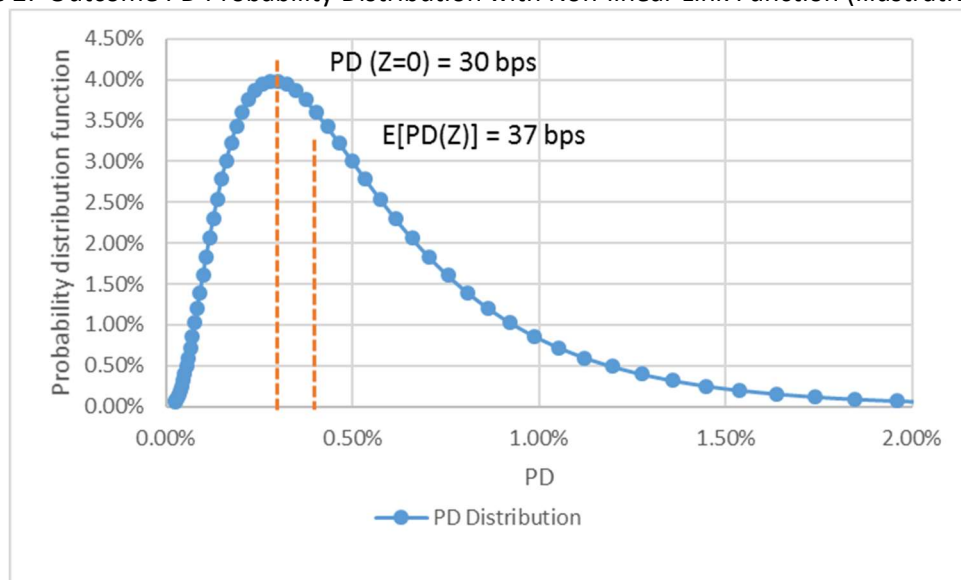


Figure 2 then shows the resultant probability distribution of PDs which shows the odds of having a heavy right tail and how the PD corresponding to the average macro-economic or credit scenario, i.e. $PD(Z=0)$, is lesser than the expected value of PD when probability weighted using distribution function of macro-economic or credit scenarios, i.e. $E[PD(z)]$.

Note that this distribution below reflects the effect only of the different possible Z values. The effect of idiosyncratic factors already are averaged in forming the Z-conditional PDs. If one specifies particular values for both systematic and idiosyncratic factors, one gets default (= 1) and non-default (= 0) outcomes, which are the observable events, rather than PDs. The PDs at each Z value in the distribution below arise as a probability weighted average of those 0/1 outcomes for every possible value of the idiosyncratic effect.

Figure 2: Outcome PD Probability Distribution with Non-linear Link Function (Illustrative data)



Moving to multi-period models^[12] and with the evolution of systematic factors exhibiting mean reversion and momentum effects, one has to use simulation in constructing the PD PDFs for future time steps and in determining the related, expected values. Also, note that the example above assumes a correlation factor of 5%, which is about the average of the values that are obtained in applying a Probit model to default data in various sectors^{[4], [5], [6], [7], [8], [9]}. With this calibration, the expected, 1-year PD of an obligor with TTC PD of 0.30% stands at about 1.2x the 1-year PD probability weighted by value of Z. If instead correlation is set to an average Basel value of 16%, the ratio of unconditional-to-conditional expectation would rise to about 1.6x. Ignoring convexity effects would be a substantial downward bias in PDs and not in line with what is expected from IFRS/CECL and Stress Testing.

This can be attributed to two reasons seen in practice:

- A. TTC nature of PD models: Most corporate and commercial models seem to be like Agency ratings and are TTC or hybrid in nature. Previous research^{[8] [9]} has demonstrated the TTC nature of Agency ratings and quantified its PIT-ness as close to 20%. If an institution's internal models are TTC or hybrid and they do not fully make it PIT using a relevant credit cycle index then they are effectively not observing the temporal variation in default rates and hence ignorant to convexity effect of the systematic factor. Since such hybrid models do not produce high and low PDs (close to observed DRs), the convexity issue becomes unobserved and hence hard to rectify. The remedy for this issue to ensure that models are fully PIT using relevant industry-region credit cycle indices as demonstrated in previous research^{[4], [5], [6], [7], [8], [9], [9]}. One such formulation of PIT PD model is described below which converts all PD models to PIT ensuring that temporal fluctuation of default rates is fully explained and model developers are able to see convexity effects in fully PIT PD models:

$$PIT_PD_{i,t} = \Phi \left(- \frac{f(Score_{i,t}) + b \cdot DDGAP_{I(i),R(i),t} + \Delta DDGAP_{I(i),R(i),t}}{\sqrt{1 - \rho_{I(i),R(i)}}} \right) \quad (2)$$

where $PIT_PD_{i,t}$ is the PIT Probability of Default for the i^{th} entity at time t
 $Score_{i,t}$ is the idiosyncratic internal model score /DD/PD for the i^{th} entity at time t
 f is a bespoke functional form for every model, e.g. logistic or Probit
 Φ - standard normal cumulative distribution function
 $DDGAP_{I(i),R(i),t}$ – industry (I) and region (R), credit-cycle index (CCI), at time t . DDGAP is a quantification of credit condition using PIT-TTC dual ratings approach. It measures how far an industry or region credit conditions are from its long run average.
 $\Delta DDGAP_{I(i),R(i),t+1}$ –change in industry (I) and region (R) credit cycle index (from t to $t+1$)
 b – regression coefficient which denotes the degree of TTC-ness of $DD_{i,t}$
 ρ – correlation factor related to $DDGAP$

- B. Desire to run fewer scenarios: Running multiple scenarios can be costly for institution and most of them have not developed industrial strength batch production systems which can produce a large number of scenarios and run end-to-end ECL calculations economically. This leads to a desire to run limited scenarios which means the convexity effect is not observed and not factored into ECL calculations. The remedy for this issue is to create batch production systems which can evaluate a large number of scenarios efficiently and can demonstrate, quantify and correct the convexity issue.

Convexity in PIT LGD and PIT EAD formulation is minor. Even though the PIT formulation has credit index in a non-linear functional form ^[9], the function is typically not convex enough in the relevant range to create substantial biases. For example, for a Tobit PIT LGD formulation ^[9], with TTC LGD (LGD (Z=0)) around 39%, the E[LGD(z)] ~ 39% and the difference is in decimal points.

3. ECL FORMULATION

Unlike Basel II RWA calculations, the ECL formulation is not specified by IFRS9/CECL. But under the conventional statisticians' definition of ECL, one is calculating the unconditional expected value of credit losses where credit loss is a random variable computed as PD x LGD x EAD.

Based on this understanding Expected Credit Losses can be defined as

$$Credit\ Loss_{i,s,t} = PD_{i,s,t}(z_{s,t}) \cdot LGD_{i,s,t}(z_{s,t}) \cdot EAD_{i,s,t}(z_{s,t}) \quad (3)$$

$$Expected\ Credit\ Loss_{i,t} = E_s[PD_{i,s,t}(z_{s,t}) \cdot LGD_{i,s,t}(z_{s,t}) \cdot EAD_{i,s,t}(z_{s,t})]$$

where $Credit\ Loss_{i,s,t}$ is a random variable representing credit loss for entity i, scenario s at time t

$Expected\ Credit\ Loss_{i,t}$ is expected credit loss for entity i, at time t over all scenarios s. The expectation (E[·]) above represents a (probability weighted) average of the product within the brackets for each of the possible Z paths. The expected value is computed by averaging results from many, multi-period, Z scenarios. In producing these scenarios, models of stochastic evolution of Zs are used.

$PD_{i,s,t}$ is a random variable representing PD for entity i, scenario s at time t

$LGD_{i,s,t}$ is a random variable representing LGD for entity i, scenario s at time t

$EAD_{i,s,t}$ is a random variable representing EAD for entity i, scenario s at time t

The PD, LGD, and EAD within the expectation on the right-hand side are conditional on survival through the end of the previous period.

Thus, the following formulation of expected credit loss is incorrect because it ignores covariance and correlation between $PD_{i,s,t}$, $LGD_{i,s,t}$ and $EAD_{i,s,t}$ terms

$$Expected\ Credit\ Loss_{i,t} = E_s[PD_{i,s,t}(z_{s,t})] \cdot E_s[LGD_{i,s,t}(z_{s,t})] \cdot E_s[EAD_{i,s,t}(z_{s,t})] \quad (4)$$

In practice, the authors have seen institutions using both formulations and, assuming a small gap between the values of the two measures, choosing the latter formulation which ignores correlation effects. In practice, this arises due to:

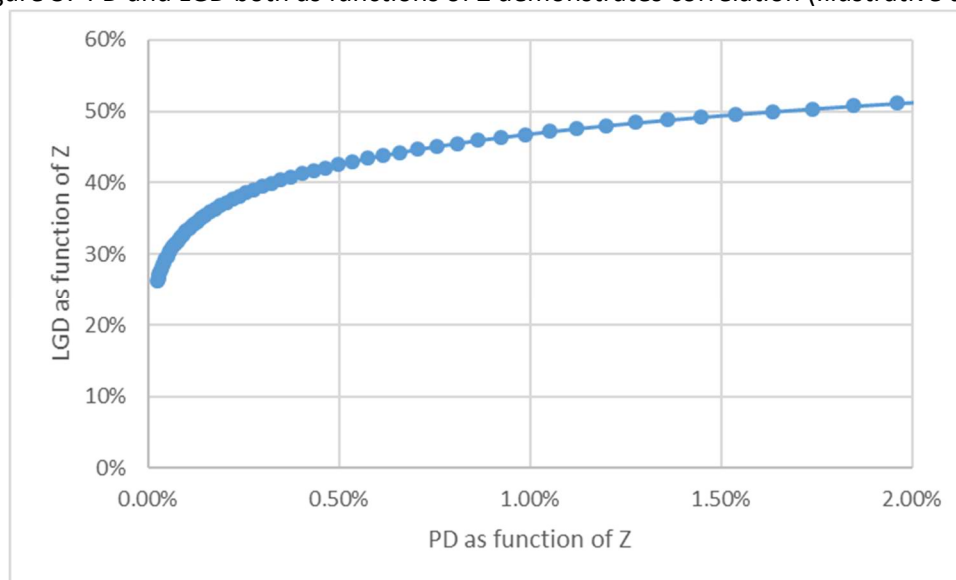
- **Hybrid TTC like models:** As discussed before, use of hybrid TTC like models in corporate and commercial modelling space would mean full temporal fluctuation of PD, LGD, EAD and as a result credit losses are understated by such models. Since these models understate the effect of credit conditions, the correlation effect gets muted and understated leading to model developers discounting and ignoring it.
- **Use of generic macro-economic indicators rather than relevant industry-region credit cycles:** Even if model developers consciously decide to derive PIT PD, LGD, EAD models, the use of generic macro-economic indicators rather than relevant industry-region credit cycles leads to a situation when correlation effect cannot be observed econometrically and hence model developers discount and ignore it. As an example, consider the use of US or UK GDP as a macro-economic driver used in PD, LGD and EAD models where there is an exposure to Oil & Gas corporations.

Since in 2015, US and UK GDP growth is healthy and above the long term norm, models built using GDP would not indicate that Oil & Gas companies are in distress (poor reflection of actual PIT risk) and further would indicate no co-movement in PD and EAD (contrary to headline news in 2015-16)

The remedy to both these issues is the usage of relevant industry-region credit cycle indices to develop PIT models which leads to full reflection of correlation in PIT PD, LGD and EAD.

Another quick fix sometimes observed in practice is use of a constant correlation factor between PD and LGD. Figure 3 shows non-linear nature of PD-LGD correlation demonstrated using the PIT PD data shown before in Figure 1 and Figure 2 and a Tobit PIT LGD (as function of Z) specification ^[9]. Clearly from Figure 3 below, using a constant value of correlation is incorrect and it would be more accurate if underlying PD and LGD values were created using credit cycle index (Z) leading to natural correlation.

Figure 3: PD and LGD both as functions of Z demonstrates correlation (Illustrative data)



4. SYSTEMATIC CREDIT CYCLES AND PD-LGD-EAD CORRELATION

There is plenty of research ^{[13], [14], [15], [16], [17], [18]} describing data, methods and insights into PD-LGD and LGD-EAD correlation and we acknowledge the existence of PD-LGD-EAD correlation. However, the approach in this study is different in the following ways:

- While previous research is focussed around Basel II and absence of correlation, this study provides insight on IFRS9/CECL unbiased probability weighted ECL which makes correlated ECL a requirement.
- Some of the previous research attempts to quantify PD-LGD and LGD-EAD correlation using data but not by describing the fundamental systematic credit cycles underlying them. This study emphasises the use of PIT measures and use of PD, LGD, EAD as functions of credit cycles (Z) which provides a solid theoretical underpinning.
- Overall past research falls short of describing and explaining PD-EAD correlation. Ample evidence indicates that such correlations exist and are best understood if one thinks of PD and EAD as

functions of credit cycles (Z). As an example Figure 5, shows evolution of Oil and Gas credit cycle index where over the past 2 years, PIT PDs have been elevated. Such a distress in Oil and Gas sector leads to higher EADs as distressed firms draw down more on existing facilities ^{[19], [20]}.

- This study emphasises the use of industry-region credit cycle indices, instead of generic macroeconomic indices to best observe and quantify correlations.
- Previous research highlights the correlation concern but does not offer an economical solution to implement correlated PD-LGD-EAD across all models in an institution. This study recommends the use of industry-region credit cycles to make such measures fully PIT and to use them consistently for all internal models (see following sections).
- This study quantifies the effect of convexity and correlation (see following sections).

5. IMPACT OF CONVEXITY AND CORRELATION ON EXPECTED CREDIT LOSSES

One gets an estimate of the effect correlation has on ECL using the same illustrative data used for Figure 1, 2, and 3. For an entity with TTC PD = 30bps, TTC LGD = 39% and a fixed EAD of \$1m, the three metrics below provide a useful quantification:

- a) Expected Loss by use of TTC PD x TTC LGD x Fixed EAD (i.e. $PD(z=0) \times LGD(z=0) \times \text{Fixed EAD}$) = \$1,184
- b) Expected Loss by use of $E[PD(z)] \times E[LGD(z)] \times \text{Fixed EAD}$ = \$1,441
- c) Expected Loss by use of $E[PD(z) \times LGD(z)] \times \text{Fixed EAD}$ = \$1,553

The increase in ECL from a) to b) is purely due to convexity effect and represents ~20% increase in ECL. As a next step, the increase in ECL from b) to c) is purely due to PD-LGD correlation and represents an ~8% increase. In reality, presence of EAD correlation would increase ECL even more. Using this illustrative data, the impact of convexity and correlation on ECL can be approximated to be 20%-30%.

The calculations are highlighted in Figure 4 below:

Figure 4: Quantifying impact of Convexity and Correlation on ECL

Z	Pr(Z)	PD	LGD	PD x LGD	EAD	ECL
<= -3	0.13%	2.21%	52%	1.14%	\$1m	
-2.9	0.19%	2.08%	52%	1.07%	\$1m	
-2.8	0.26%	1.96%	51%	1.00%	\$1m	
.	.	.	.	Ignoring Convexity and Correlation leads to 20%-30% lower ECL		
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-0.1	3.94%	0.32%	40%	0.13%	\$1m	
0	3.98%	0.30%	39%	0.12%	\$1m	\$ 1,185
0.1	3.98%	0.28%	39%	0.11%	\$1m	
.	.	.	.	.	.	
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.	.	.	.	.	.	
2.8	0.07%	0.03%	27%	0.01%	\$1m	
2.9	0.05%	0.03%	27%	0.01%	\$1m	
>=3	0.13%	0.02%	26%	0.01%	\$1m	

Pr Wt Average	0.37%	39%		\$1m	\$ 1,442
Pr Wt Average			0.16%		\$ 1,554

PD Convexity Effect is material

LGD Convexity Effect is immaterial

ECL Convexity Effect is material

ECL Correlation Effect is material

6. EFFICIENT WAY TO INCORPORATE CONVEXITY AND CORRELATION ACROSS MODELS

Now that hopefully that the reader is convinced that convexity and correlations exist and are important, the authors propose remedial action by incorporation of industry-region credit cycle indices in internal models of an institution. This addresses two concerns:

- Use of correct industry-region credit cycle index rather than a generic macro-economic time series which leads to failure to observe correlation in certain models/portfolios,
- Use of consistent credit cycle index across model types and asset classes to do this in a cost effective way otherwise firms will consider it an unreasonable cost.

A typical small bank with diversified corporate/commercial lending book could have 10+ models and a large wholesale bank could easily have 40+ models. This poses the question; how can one come up with single source of data for accurately creating PIT measures and hence incorporating correlation across model types (PD, LGD, EAD) and across asset classes (Corporates, Banks, Funds, etc.)?

Although studies exist on correlation across corporate portfolios, such studies are typically not customised to a financial institution's portfolio, type of models and not across all model types. Hence use of such correlation studies is limited to evidence of correlation, and at best, benchmarking.

In the authors' experience, Industry-Region credit cycle indices derived from sources such as Moody's KMV [4], [5], [6], [7], [8] [9] and [9] are a rich source of consistent credit cycle index data which can be used across model types and asset classes. The table below provides a quick overview of several such model types and the type of credit indices used for PIT model formulation.

Table 1 Use of Industry-Region Credit Cycle Indices across models for consistent quantification of correlation

Models	Type of Credit Cycle Index Used across PD, LGD, EAD models
Corporates	Mixed Industry and Region
Banks	Region specific or Global Banking sector
Property	Region specific Real Estate sector
Project Finance	Mixed Industry and Region
Funds	Region specific or Global Financial Institutions sector
Insurance	Region specific or Global Insurance sector
Broker Dealers	Region specific or Global Insurance sector
Shipping	Global Transportation sector

These credit cycle indices are derived from summarizing, within selected industries and regions, PDs from a broad-based, fully PIT model such as Moody's CreditEdge™. These indices are used as conditioning factors in models for deriving PIT PD, LGD and EAD probability distribution function (PDF). Different variants of industry and region credit cycle indices allows one to recognize differences in the credit conditions of different sectors [9].

With models for all PIT PD, LGDs and EADs in hand, one now can now apply them along with models for PIT PDs in creating joint, PD, LGD, and EAD scenarios and thereby PIT loss scenarios. The topic and sub-topics of deriving PIT loss scenarios deserve detailed sections in themselves but are summarized in previous research [9] and here briefly. One may accomplish the task of deriving PIT losses by:

- developing Z-factor scenarios in one of the ways discussed below,
- entering the one-period Z changes (ΔZ) from each Z-factor scenario into a multi-threshold, Probit model of transitions from each credit state (grade) to default and to each, non-default state and thereby creating, for each entity with known, initial, PIT PD scenarios,
- entering the Z values from each Z-factor scenario period-by-period into the PIT LGD and EAD models and thereby obtaining PIT, ELGD (Expected LGD) and EEAD (Expected EAD) scenarios that are jointly determined with the PD ones.

From each joint, PD, ELGD, EEAD scenario, one gets an ECL scenario by forming the period-by-period products: $\Delta PD \times ELGD \times EEAD$. Note that PD denotes a cumulative probability of default and ΔPD represent a one-period, marginal probability of default.

To generate period-by-period PD scenarios from the repeated application of one-period, PIT PD models, one must create scenarios for the one or more inputs into those models. Perhaps the simplest way to accomplish this involves the Probit, credit-factor-conditional, transition-matrix model. This was first introduced to a broad audience by CreditMetrics [12] and it now serves as a mainstay of the regulatory-stress-test processes at many institutions.

One can view the grades or credit states in the model as binned default-distance (DD) measures, with DD representing the single, consolidated input into a Probit PD model. The transition rates to default reflect the PIT PD model. The transition rates to the other states correspond to the stochastic projections of PD-model inputs. Thus, the transition model includes both a PD model and an input-evolution model and so amounts to a tidy way of consolidating all of the needed components. Observe that the transition probabilities vary with ΔZ , which is the one-period change in Z . PIT transitions involve changes in PIT states. Thus, changes in cycle rather than absolute level of the cycle influence transition rates. In contrast, levels of the cycle influence the ELGDs and EEADs. The process of determining ECL scenarios for stress testing and for IFRS 9/CECL reduces to the generation of Z scenarios. Here one has two broad options:

Statistical scenarios: Surely the simplest way of producing statistical scenarios involves the use of time-series models. These models are commonly used in credit-value-at-risk applications.

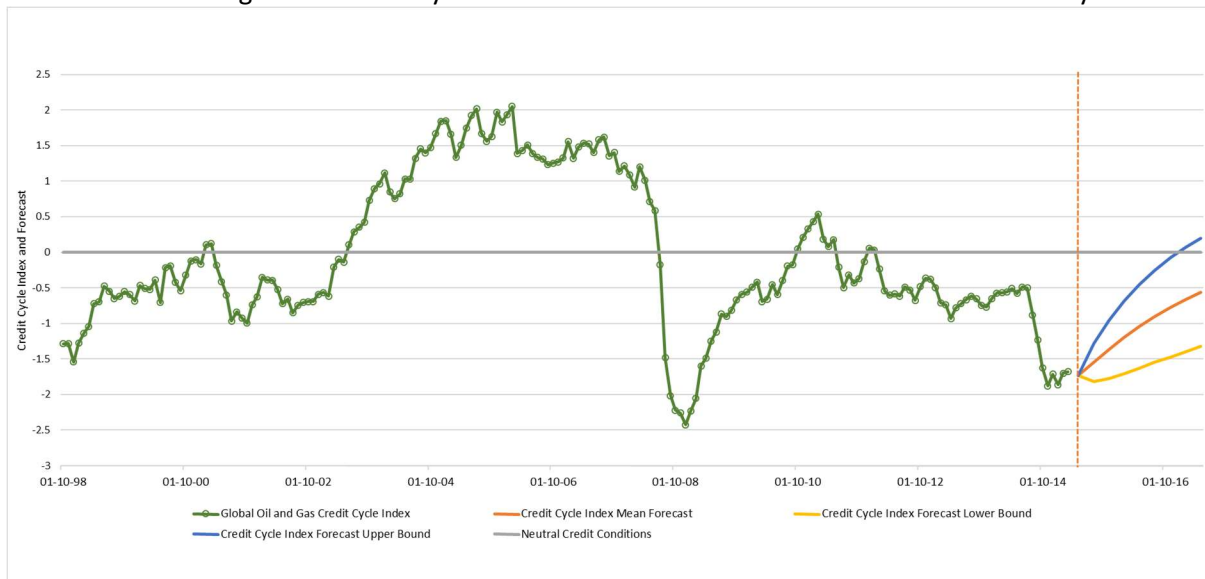
One starts with the current and past values of the relevant Z s and obtains scenarios for their stochastic evolution by applying AR1 or AR2 models calibrated to historical experience. In this case, the Z factors evolve under the influence of

- i) mean reversion, which causes them to gravitate toward zero,
- ii) momentum, which pushes them in the same direction as they moved most recently, and
- iii) most importantly, random shocks.

One can quite easily estimate such models and apply them in generating thousands of scenarios. By averaging the ECLs implied by the joint PD, ELGD, and EEAD scenarios arising from such Z scenarios, one gets a result corresponding to the probability weighted average, ECL term structure that IFRS 9 or CECL demand.

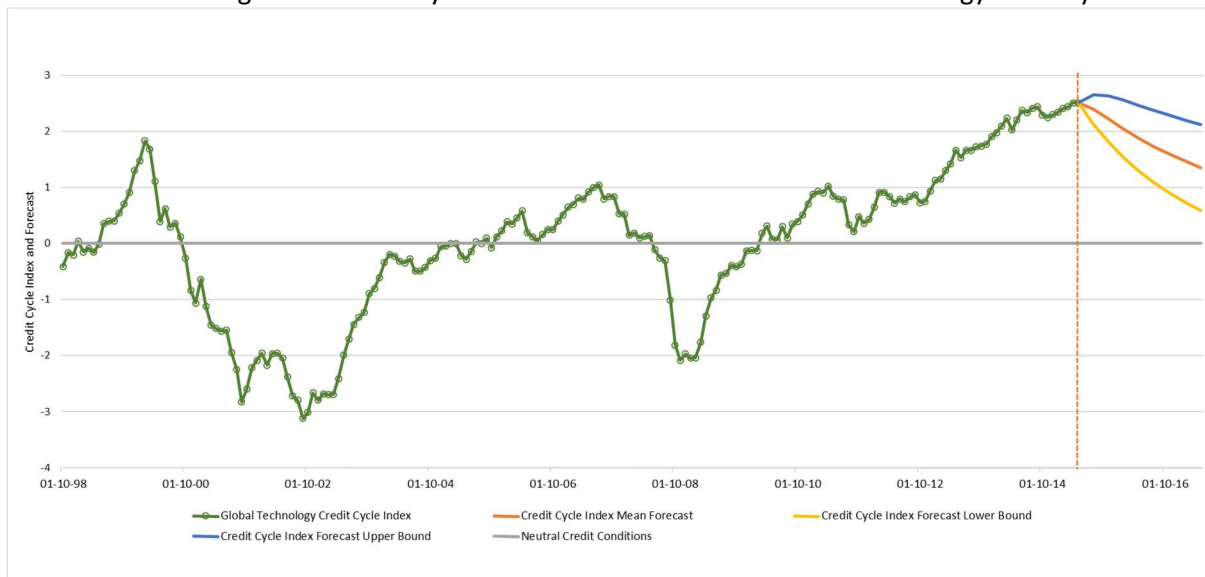
See below Z paths for two sectors with very different initial conditions and hence quite different outlooks (Figure 5 and Figure 6). In each case, momentum has a short run effect and mean reversion a persistent one. Stochastic projections for the next 8 quarters are demonstrated below. Although only three lines (mean, 95th percentile, and 5th percentile) corresponding to the period-by-period paths are shown in Figure 5, in practice thousands of simulations of $Z_{t, t \rightarrow \text{today}}$ which then drive simulated LGD_t paths are drawn using Monte Carlo Simulation techniques.

Figure 5: Credit Cycle Index and Forecasts for Global Oil and Gas Industry



Sources: Moody's Analytics CreditEdge™, Z-RiskEngine™

Figure 6: Credit Cycle Index and Forecasts for Global Technology Industry



Sources: Moody's Analytics CreditEdge™, Z-RiskEngine™

Deterministic scenarios: In this case, one assumes that a particular scenario occurs and then derives the credit outcomes implied by it. One may perform this exercise for a handful of distinct scenarios representing, for example, baseline, stress, and severe stress conditions. If these exercises involve pre-defined credit-factor paths, such as Z paths, then the process of determining credit outcomes is straightforward. One merely enters the ΔZ paths into the transition model and the Z paths into the LGD and EAD models. However, in many cases including most importantly the regulatory stress tests, the predetermined scenarios specify paths not for aggregate-credit factors but for macroeconomic variables. Unfortunately, most of the predetermined macroeconomic variables that the regulators specify have only tenuous relationships to the wholesale credit cycle. Thus, one must develop a further model bridging from the prescribed macroeconomic variables to the credit factors with proximate effects on PDs, LGDs,

and EADs. Building a credible bridge model is a non-trivial task and deserves a much lengthy discussion in itself, but such models can be easily built if one has an existing Stress Testing model which does some sort of bottom-up loss forecasting.

One can imagine using the bridge model along with several macroeconomic scenarios as an alternative to the time-series approach to estimating unconditional ECLs. There are few, macroeconomic models available to credit institutions and none, to the authors' knowledge, have been used recently in developing large numbers of statistical scenarios. With regard to judgmental scenarios, they have poor track records in part on account of psychological biases.

In closing, the authors observe that, to estimate the probability weighted average of a facility's lifetime credit losses accurately, one must account for the non-linear response of losses to the credit cycle. This non-linear response reflects two phenomena: the convexity of PD functions in the relevant range; and positive PD, LGD, EAD correlation. Only by averaging the ECLs arising from many, joint, PD, LGD, EAD scenarios does one account for these non-linear effects.

7. SUMMARY

In this study, the authors demonstrate the presence of convexity and correlation effects which impact ECL. For the convexity effect, the impact on ECL is highly dependent on the PD function's convexity along with the amount of random dispersion in the credit-risk factors affecting PDs. For the correlation effect, the impact on ECL is dependent on the magnitude of PD-LGD and PD-EAD covariance. Hence, to calculate ECL accurately, one must account for both these effects. Using illustrative data, the impact of convexity and correlation on ECL is an upward correction of 20% or more which is important given the regulatory and accounting requirements of unbiased measure of ECLs. As a remedy, the authors recommend use of PIT PD, LGD, EAD models conversion using relevant industry-region credit cycle index. Once, an institution has fully PIT models using relevant credit cycle index, a correct ECL formulation can capture both the convexity and correlation effects and lead to unbiased measure of ECLs required for IFRS9/CECL and Stress Testing.

DECLARATION OF INTEREST

The authors report no conflicts of interest. The authors alone are responsible for the content and writing of the paper.

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