Expect the unexpected

The stakes are high in the quest for efficient transaction-level measures of loan credit risk. Barry Belkin, Larry Forest, Scott Aguais and Stephan Suchower investigate the properties of the premiums payable for "insuring or "swapping" such risks

Financial institutions have a high stake in the quest for better measures of credit risk, as such measures are fundamental to their efforts to account properly for credit risk in both pricing and portfolio management.

In this article, we develop a direct approach to measuring credit risk at the transaction level. Under this approach, we identify credit risk as a "cost" – most simply, the cost of buying credit "insurance" that transfers the risk to the seller, after the approach presented in Merton & Bodie (1992) and Merton & Perold (1993). This approach emphasises that every credit instrument has an associated risk premium, which corresponds to the competitively determined cost of a form of credit insurance. In what follows, we develop the basic risk premium concept and provide some simple numerical examples.

In describing credit risk, analysts have attached different meanings to the term "risk premium". In some contexts, it denotes a payment compensating for both the expected or actuarial value of losses associated with an instrument and the (downside) volatility that the instrument contributes to a diversified investment portfolio.

In other contexts, analysts view only the second of these components as representing risk. They then use the term more narrowly, in referring just to the uncertain component of credit loss. For clarity, we will label the first component the expected loss (EL) risk premium, and the second the unexpected loss (UL) risk premium (see Ginzburg, Maloney & Willner, 1994).

In discussing casualty and other forms of commercial insurance, a "premium" represents a cost paid by the policy-holder to transfer the risk of loss to the issuer of the policy. Most often, the premium covers both expected and unexpected loss as described above.

This insurance perspective helps in understanding credit derivatives. The counterparty acquiring risk in a credit derivative transaction essentially sells credit insurance. The cost of this insurance constitutes the risk premium. In the case of a credit default swap, the party ceding risk is in essence purchasing a constant-premium term insurance policy covering credit loss on a specific loan. As an alternative, the financial institution could purchase a put option on the credit loss, in which case it would make a single upfront payment for the protection. We then view the upfront payment as an option premium.

If a commercial financial institution chooses to hold an exposure and thus "self-insure" against credit loss, some portion of the loan revenue must implicitly go to support the risk capital held by the institution to cover its unexpected credit losses. The provisioning process covers expected credit loss. In this context, one can think of a bank's commercial lending business internally transferring a portion of its loan revenue into a risk capital account. Distributions from this account provide bank shareholders with a return on their risk capital. As the majority of the credit risk is borne by the shareholders, the return on their risk capital amounts to a risk (insurance) premium paid by the commercial lending line of business.

We will define the risk premium associated with a credit instrument in a way that focuses on the cost of swapping unexpected total return (UTR), ie, deviations from expected total return, as opposed to the cost of insuring against unexpected default loss (UDL). But before we get to the specific concept, we need to lay some groundwork. Specifically, we need to distinguish between the "natural" probability measure governing credit rating migration and the associated "risk-neutral" migration measure.

Natural and risk-neutral measures

We assume that the borrower can be assigned a discrete credit rating that represents a sufficient statistic for predicting credit losses. This could come from a public ratings agency or from the lender's internal risk-rating system. We refer to the probability measure that governs the migration of the borrower's risk rating over time as the natural process measure.

The theory of risk-neutral valuation specifies that, under certain conditions, contingent claims can be priced uniquely by arbitrage (see Harrison & Pliska, 1981). In particular, and most importantly, the instrument(s) underlying the contingent claim must trade in a market free of arbitrage opportunities.

In the current context, that market is the secondary commercial loan market. In the past, loans were generally regarded as "buy-and-hold" assets for originators. In recent years, however, trading volumes and liquidity in the large corporate market have improved substantially, making the arbitrage-free assumption more plausible. A discussion of the justification and detailed methods for applying arbitrage pricing theory to commercial loans and credit derivatives would take us too far afield; we refer the interested reader, therefore, to Ginzberg, Maloney & Willner (1994), and Belkin, Suchower, & Forest (1998).

In any case, our point of departure is that arbitrage pricing theory applies to credit markets and provides an internally consistent and empirically supported framework for pricing. Under this hypothesis, one may calculate the arbitrage-driven market price of a commercial loan or credit derivative as the expected net present value of the cashflows it generates. This expectation calculation, however, needs a particular probability measure.

We do not use the natural process measure, but rather a related measure, technically referred to as the (unique) equivalent martingale process measure, but more commonly known as the risk-neutral process measure. We again refer the reader to the two references immediately above for a detailed discussion of the formal procedure for constructing the risk-neutral migration measure associated with a given natural migration measure.[1]

In what follows, we use the notation $E[X]$ to denote the expectation of the random variable X with respect to the natural migration measure, and $\tilde{E}[X]$ to denote the expectation of X with respect to the risk-neutral measure.

Defining credit risk premiums

We take as the definition of unexpected total return (UTR) risk premium:

$$\text{UTR risk premium} \stackrel{\text{def}}{=} E[V] - \tilde{E}[V] \quad (1)$$

where V denotes the net present value (NPV) of the credit instrument cashflows to the lender. The possibility of credit loss makes V a random variable.[2] In general, the discounting used in calculating V involves the risk-free short rate of interest, r. If one models interest rates as having a stochastic term structure, then the expectation operators in (1) will reflect the joint interest rate and credit migration process. In this context, we restrict our attention to the simple case in which the short rate is known deterministically.

The UTR risk premium, as defined in (1), is the difference between the NPV that the lender expects to realise by holding the credit instrument for its cashflows and the price that it would obtain by selling the instrument at its (arbitrage-determined) fair market value. The UTR risk premium therefore represents the lender's economic incentive to bear the risk associated with holding the instrument.

To gain some insight into this particular definition of the risk premium, we consider a commercial lender that manages its activity using two separate accounts: an operating account, which processes cashflows, and a capital account, where profits and losses normally occur. The capital account receives a risk premium paid out of the operating account. In exchange, the capital account makes payments into the operating account to offset any "unexpected" loan revenue $V–E[V]$ (see table A).

A. Operating account and capital account cashflows

	Front-end UTR risk premium	Loan revenue	Back-end settlement	Net cashflow
Operating account	$-(E[V] - \tilde{E}[V])$	V	$E[V] - V$	$\tilde{E}[V]$
Capital account	$E[V] - \tilde{E}[V]$		$-(E[V] - V)$	$V - \tilde{E}[V]$

Table A shows that the net cashflow into the operating account has present value $\tilde{E}[V]$, independent of the actual loan revenue V. The effect is therefore equivalent to the sale of the loan at its fair (arbitrage-determined) market value. The net cashflow into (or out of) the capital account is $V - \tilde{E}[V]$, so the capital account has effectively purchased the loan at its market value.

One perspective on the transaction is that, by paying the risk premium, the operating account enters into a form of credit swap with the capital account, exchanging a risky set of cashflows with return V for a riskless set of cashflows with return $\tilde{E}(V\}$. Two cash exchanges occur: (i) the front-end payment by the operating account to the capital account of the risk premium; and
(ii) the back-end settlement between the two accounts to offset the unexpected loss/gain in the operating account. Note that the expected value of the back-end settlement is zero.

It is important to distinguish between the UTR risk premium, which is really the cost of a credit swap, and the UDL risk premium, which is the premium on a default loss "insurance policy".

In the case of the swap, the financial institution is transferring both the upside and downside risks of any unexpected deviation in total return. In the default-loss insurance case, the financial institution is insuring specifically against the one-sided risk of default loss in excess of expected default loss.

We will show below that, in general, the premiums for UTR risk transfer and UDL insurance are different.

Properties of credit risk premiums

In order to motivate our discussion of credit risk premiums further, in what follows we summarise their primary properties. First, if the cashflows associated with a credit instrument are deterministic, the associated risk premium should be zero. We can see that this condition holds for our definition because, if cashflows are riskless, the natural measure and risk-neutral measure are equivalent (ie, have the same zero probability events). If only one particular outcome is possible under a given measure, then only that same outcome is possible under an equivalent measure. Consequently, if there is no risk, there can be no risk premium.

We also observe that risk premiums can be negative. Consider the situation in which a lender holding a high-risk loan enters into a credit default swap to hedge its credit risk. From the lender's perspective, the loan has a positive risk premium because it creates a credit risk exposure, while the credit default swap has a negative risk premium because it offsets credit risk. Of course, from the standpoint of the credit default swap counterparty, which now holds the default risk, the deal has a positive risk premium.[3]

One might reasonably postulate that the loan risk premium and the credit default swap risk premium will offset each other exactly. As the numerical examples

[1] Briefly, one starts with the risk-neutral measure for one-period (option-free) term loans, which are effectively priced through observed market credit spreads. Arbitrage pricing methods can then be used to price two-state (default or non-default) "reference loans" with arbitrary payoffs by treating such instruments as contingent claims on one-period loans. The prices of general multi-state, multi-period loans are obtained recursively by approximating their one-period cash flows in terms of these reference loans. The numerical examples in this article are based on a spreadsheet program which implements this procedure specifically for two-period loans.

[2] Of course, there are many potential sources of randomness in the present value of cash flows generated by a commercial loan in addition to credit loss. In the case of a fixed rate loan, there is variability in interest rates. Facility utilization is an important source of variability in a revolving facility.

[3] It is perhaps tempting to think of a negative risk premium as indicative of "anti-risk," which negates risk. However, risk to the party on one side of a credit derivative transaction is anti-risk to the party on the other side. Consequently, one cannot really distinguish between risk and anti-risk. The choice of algebraic sign to attach to a risk premium is really a matter of whose perspective one takes when calculating net present value. We observe that in principle a lender should never hold a position with respect to a given borrower that results in a net negative risk premium. The lender would do better selling the position to a counterparty in search of "anti-risk" to reduce his credit exposure to that same borrower.

below demonstrate, however, this will not generally be the case, and so the net position (loan plus credit default swap) will not always be perfectly hedged. The loan spread and the swap spread may differ. In that case, the lender acquires protection against default loss, but not against the basis risk inherent in the spread differential.

Net present value is "additive" in the restrictive sense that the NPV of two pooled cashflows tied to the same borrower rating migration process is the sum of the NPVs of the component cashflows. Furthermore, expectation is a linear operator. Consequently, one can calculate the risk premium for a hedged position relative to a single credit exposure as the sum of the risk premiums of the separate instruments that comprise the hedge.[4]

We next show that risk premiums, as we have defined them, generalise the notion of marginal risk premiums. Consider a one-year term loan made to a borrower whose rating grade at the loan origination is i, and let:

- pD(i) be the probability of default;
- LIED (loss in the event of default) be the fraction of the loan balance that is not recovered in the event of default; and
- $\phi(i)$ be the par credit spread on the one-year loan if the borrower risk grade at loan origination is i.

In this context, we will treat LIED as deterministic. We also assume that any interest payment due at the time of default is paid in full. We now define the marginal risk premium u(i) associated with a loan originated to a borrower with risk grade i as

$$u(i) = \phi(i) - p_D(i)\text{LIED} \tag{2}$$

Thus, u(i) represents the excess of the par credit spread over the expected default loss, where we have calculated expected default loss with respect to the natural migration measure. The term marginal here indicates that u(i) represents the incremental risk premium earned by the lender per \$1 of loan principal if the term of the loan is extended by one year.

Under the risk-neutral measure, the one-year default probability $\pi_D(i)$ becomes (see Ginzburg, Maloney, & Willner, 1994):

$$\pi_D(i) = \frac{\phi(i)}{\text{LIED}} \tag{3}$$

It then follows that

$$\begin{aligned} E[V] - \tilde{E}[V] &= \left(1 - p_D(i)\text{LIED}\right) \\ &\quad - \left(1 - \pi_D(i)\text{LIED}\right) \\ &= \phi(i) - p_D(i)\text{LIED} \\ &= u(i) \end{aligned} \tag{4}$$

Thus, in the special case of a one-year loan, the UTR risk premium reduces to the marginal risk premium. For the same one-year loan, the UDL risk premium is given by:

$$\begin{aligned} &= \pi_D(i)\left(1 - p_D(i)\right) \cdot \text{LIED} \\ &= \phi(i)\left[1 - p_D(i)\right] \end{aligned} \tag{5}$$

A comparison of (4) and (5) shows that the UDL risk premium and UTR risk premium differ by $p_D(i)[\text{LIED}-\phi(i)]$. As the par credit spread $\phi(i)$ is by necessity less than LIED, this difference is a positive quantity. We shall see below, however, that the UDL risk premium is not always larger than the UTR risk premium for multi-period loans.

Multi-period loans

To investigate the properties of risk premiums for multi-period loans, we have constructed a spreadsheet that calculates the expected present values for the cashflows of a simple (option-free) two-year term loan. We assume a rating system with eight possible risk grades – Aaa, Aa, A, Baa, Ba, B, Caa and default – and assume the average one-year migration matrix, conditional on no rating withdrawal given for the period 1920–96 in Carty (1997). In order to retain our focus on risk, we ignore the costs of loan origination or of carry (servicing and monitoring). Our results reflect the case LIED=0.40 and the risk-free rate r=0.045 per annum. The assumed marginal risk premiums (taken to have a flat term structure) and the resulting par credit spreads are shown in table B.

The one-year par credit spreads are obtained for each rating by adding together the expected loss component and the unexpected loss (marginal risk premium) component. Observe the term effect that the two-year par credit spreads are higher than the one-year par credit spreads for every initial rating except Caa. The reason is that if a Caa credit does not default in year one, the migration matrix predicts a 0.092 conditional probability of an upward migration and a corresponding reduction in the second-year exposure relative to the first-year exposure. For all other ratings, the risk of a downward migration in year one dominates that of an upward migration and the second year exposure is higher than the first.

B. Par credit spreads

	Risk rating at loan origination						
	Aaa	**Aa**	**A**	**Baa**	**Ba**	**B**	**Caa**
One-year expected default loss	0.4bp	2.8	5.6	12.4	50.0	154.8	552.4
Marginal risk premiums	0.1bp	0.3	1.1	2.2	10.0	45.0	110.0
One-year par credit spreads	0.5bp	3.1	6.7	14.6	60.0	199.8	662.4
Two-year par credit	0.7bp	3.3	7.2	16.4	63.9	202.8	642.4

C. Comparison of unexpected total return risk premium with unexpected default loss risk premium

	Risk rating at loan origination						
	Aaa	**Aa**	**A**	**Baa**	**Ba**	**B**	**Caa**
UTR risk premium	\$0.23	\$0.66	\$2.21	\$4.79	\$20.46	\$82.62	\$187.85
UDL risk premium	\$0.23	\$0.66	\$2.21	\$4.78	\$20.32	\$80.54	\$171.75

We first compare the UTR and UDL risk premiums as a function of the initial rating for the two-year loan with an assumed principal of \$10,000. The results are shown in table C.

As one would expect, the loan risk premiums are quite small for low-risk borrowers, but increase significantly as the credit quality of the borrower deteriorates. At an Aaa borrower risk rating, the UTR risk premium and UDL risk premium are each about 0.2 basis points relative to the \$10,000 loan principal. For the Caa rated borrower, however, both risk premiums increase dramatically to exceed 170bp.

Transferring unexpected total risk for a two-year loan commands a premium at least as large as that required to insure only unexpected default-loss risk. This contrasts with the case of a one-year loan, where we showed that the opposite relationship holds.

The explanation lies in the fact that the only risk in a one-period loan is that of default-loss risk (the first year interest is as-

[4] *Risk premium additivity holds in the present context precisely because the only risk is that of borrower default. Risks of different types (for example, interest rate risk and default risk) do not add. Consequently, the risk premium for a hedged position consisting of a fixed rate loan, an interest rate swap (exchanging fixed rates for floating rates), and a credit default swap is not the sum of the risk premiums of the component instruments. The basic notion of risk premium nonetheless remains valid for such a hedged position, provided the required expectations are calculated with respect to the natural measure and the risk-neutral measure for the joint interest rate and rating migration process.*

sumed effectively to be prepaid). If the financial institution buys unexpected default-loss insurance, it keeps the unexpected gain when default does not occur. If the financial institution enters into an unexpected total return swap, it must pay that unexpected gain to the counterparty on settlement. As a result, the UTR risk premium is higher on the one-period loan than the UDL risk premium.

In the case of a two-year loan, both the return of principal and the second-year interest payment are at risk. A UTR swap protects against both of these risks. UDL insurance protects only against the loss of loan principal. The results indicate that the extra protection afforded by the UTR swap more than offsets the settlement cost if the loan matures without default. The differential between the risk premiums is barely discernible at the higher rating grades, but becomes increasingly significant as one progresses down the rating scale.

Loan pricing

We next examine the effect of loan pricing on the loan risk premium. The technique of utilising credit risk premiums detailed in this article is the foundation upon which credit risk is measured in KPMG's Loan Analysis System.

In what follows, all references to risk premiums will be to UTR risk premiums. We assume a loan of $10,000 principal to a Caa rated borrower and determine the risk premium for plus or minus 100bp variations in the loan spread relative to par. The results are given in table D.

In the case of a Caa rated borrower, the effect of the spread on risk premium is about 2.5bp of risk premium per 100bp of spread variation. Other tests demonstrate that the effect of the spread on risk premium rapidly diminishes with increasing risk grade and all but vanishes at the investment-grade end of the risk spectrum.

Next, we explore the effect of hedging on risk premiums. We consider three alternative lender strategies:

(i) hold only the loan and bear the full credit risk;

(ii) enter into a credit default swap (at par) to hedge the credit risk on the loan; or

(iii) purchase a default put to hedge the loan risk.

In the case of the credit default swap, the holder of the loan pays a fixed spread to the counterparty to insure against credit loss. The spread is paid annually until the loan either matures or defaults, ignoring the possibility of prepayment by the borrower. In the current context, we use the term "put" to refer to the case in which the holder of the loan makes a single upfront payment to the counterparty to purchase credit loss insurance for the term of the loan.

In table E we show the risk premium for each of the exposures for all three of these alternatives, and for each borrower risk rating at loan origination. Both the loan and the credit default are assumed to be priced at their par spreads under risk-neutral pricing. The default put is priced at its risk-neutral market value. Again, the loan principal at origination is taken to be $10,000.

The [loan plus default swap] position has a zero risk premium independent of the borrower rating. This is as one would expect, because the par spread on the loan and the par spread on the credit default swap are equal. The spread the lender earns from the borrower exactly offsets the payments the lender makes to the swap counterparty. In addition, any loss of principal resulting from borrower default is offset by a payment from the swap counterparty. The net effect is that the lender simply earns the risk-free rate in all cases.

The (loan plus default put) position is an incomplete hedge, as the cashflows have some residual degree of risk. If the borrower defaults, the lender is protected against any loss of principal. However, if the loan defaults in year one, the lender loses the interest that would have been paid in year two. As table E indicates, this risk of loss of loan income is very much a function of the borrower's risk rating.

A comparison of tables C and E shows that the UTR risk premium for the (loan plus default put) hedge is equal to the excess of the UTR risk premium on the loan over the UDL risk premium on the loan. This follows from a combination of:

(i) the fact that the UDL risk premium on the loan is the same as the UTR risk premium on the default put; and

(ii) the applicability of risk premium additivity.

Conclusion

In summary, the point of view we have taken is that the credit risk associated with a credit instrument can be measured by the cost for the holder to enter into a form of swap that transfers to the counterparty the risk of any variation (positive or negative) between the actual return that instrument generates and its expected return. Credit risk can be self-insured – in which case the "counterparty" is the holder's risk capital – or swapped to a true counterparty through a form of credit derivative. In either case, the risk premium is what the holder must pay at arbitrage-free market pricing to effect the complete transfer of the credit risk.

In a follow-up article, we will develop in greater detail the relationship between credit risk premiums and risk capital. We will show how that relationship leads to a method for estimating risk-adjusted returns at the transaction level and provides a natural link between transaction credit risk and portfolio VAR. ■

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D. Effect of load spread on risk premiums

Loan principal $10,000
Borrower risk rating Caa

Loan spread (bp)	Risk premium
par–100=542.4	$185.34
par=642.4	$187.85
par+100=742.4	$190.36

E. Risk premiums for different hedged positions

	Risk rating at loan origination						
	Aaa	**Aa**	**A**	**Baa**	**Ba**	**B**	**Caa**
Loan	$0.23	$0.66	$2.21	$4.79	$20.46	$82.62	$187.85
Loan + default swap	0	0	0	0	0	0	0
Loan+ default put	<$0.01	<$0.01	<$0.01	$0.01	$0.14	$2.08	$16.10

REFERENCES

Merton, RC, and Z Bodie, 1992, *On the management of financial guarantees*, Financial Management, Winter

Merton, RC, and A Perold, 1993, *Management of risk capital in financial firms*, in "Financial services: perspectives and challenges", ed. Samuel Hayes, Harvard Business School Press

Ginzburg, A, KJ Maloney and R Willner, 1994, *Debt rating migration and the valuation of commercial loans*, Citibank Portfolio Strategies Group Report, December

Harrison, J M, and SR Pliska, 1981, *Martingales and stochastic integrals in the theory of continuous trading*, Stochastic processes and their applications, 11, pages 215–260

Belkin, B, SJ Suchower and LR. Forest Jr, 1998, *The effect of systematic credit risk on loan portfolio value-at-risk and loan pricing,* CreditMetrics® Monitor, First Quarter, pages 17–28

Carty, L, 1997, *Moody's ratings migrations and credit quality correlations, 1920–1996,* Moody's Investors Service Report